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CORPORATE MERGERS, INVESTMENT INCENTIVES AND FIRM VALUE: AN AGENCY THEORETIC ANALYSIS

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CORPORATE MERGERS, INVESTMENT INCENTIVES AND FIRM VALUE:

AN AGENCY THEORETIC ANALYSIS

Teresa A. John

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1985

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Teresa A. John

Sworn and subscribed to before me this 25th

DAY OF October 1985.

C. Marie Mae

My Commission Expires March 23, 1986

PREFACE

A special thanks to my chairman, Joshua Ronen, for his invaluable help and encouragement. Thanks also to my committee, K. R. Balachandran, Michael Keenan, and Joel Owen, and also to John Bildersee, Dov Fried, Ernest Kurnow and Itzhak Swary for their support and encouragement.

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SUMMARY OF PROPOSITIONS AND COROLLARIES

Chapter IV: Financial Synergy of Mergers

Proposition I:

(a) $s_1^\circ = s_2^\circ$ if and only if $s_m^\circ = s_1^\circ = s_2^\circ$.

(b) $s_1^\circ \neq s_2^\circ$ (w.l.o.g. $s_1^\circ < s_2^\circ$) if and only if $s_1^\circ < s_m^\circ < s_2^\circ$.

Corollary 1.1:

(a) $s_1^\circ = s_2^\circ$ if and only if $\Delta A_1 = \Delta A_2 = 0$.

(b) $s_1^\circ < s_2^\circ$ if and only if $\Delta A_1 > 0$, $\Delta A_2 < 0$.

Corollary 1.2:

Given $\hat{s}_1 = \hat{s}_2$,

(a) $s_1^\circ = s_2^\circ$ if and only if $D_1 = D_2$.

(b) $s_1^\circ < s_2^\circ$ if and only if $D_1 < D_2$.

Corollary 1.3:

Given $\hat{s}_1 = \hat{s}_2$,

(a) $D_1 = D_2$ if and only if $\Delta A_1 = \Delta A_2 = 0$.

(b) $D_1 < D_2$ if and only if $\Delta A_1 > 0$, $\Delta A_2 < 0$.

Corollary 1.4:

Given $\hat{s}_1 = \hat{s}_2$,

(a) $s_1^\circ = s_2^\circ$ if and only if $U_1 = U_2$.

(b) $s_1^\circ < s_2^\circ$ if and only if $U_1 < U_2$.

Corollary 1.5:

Given $\hat{s}_1 = \hat{s}_2$,

(a) $U_1 = U_2$ if and only if $\Delta A_1 = \Delta A_2 = 0$.

(b) $U_1 < U_2$ if and only if $\Delta A_1 > 0$, $\Delta A_2 < 0$.

Proposition 2:

(a) If $s_1^\circ = s_2^\circ$, then $\Delta V = 0$.

(b) If $s_1^\circ \neq s_2^\circ$ (w.l.o.g. $s_1^\circ < s_2^\circ$) and $\hat{s}_1 \geq \hat{s}_2$, then $\Delta V > 0$.

Corollary 2.1:

Given assumptions (A1) to (A9), a merger will always result in a non-negative increase in value ($\Delta V \geq 0$). Further, if $D_1 \neq D_2$ (or if $U_1 \neq U_2$), a merger will always result in a positive increase in value ($\Delta V > 0$).

Chapter V: Wealth Transfers among Claim Holders

Proposition 3:

(a) $s_1^\circ = s_2^\circ$ if and only if $\Delta E = 0$.

(b) $s_1^\circ \neq s_2^\circ$ (w.l.o.g. $s_1^\circ < s_2^\circ$) if and only if $\Delta E < 0$.

Corollary 3.1:

(a) $\Delta B = \Delta V + |\Delta E|$.

(b) If $\Delta V \geq 0$, then $\Delta B \geq 0$.

(c) If $\Delta V > 0$, then $\Delta B > 0$.

Proposition 4:

Given $s_1^\circ < s_2^\circ$, $\Delta B > 0$ if and only if $\frac{F_1}{b_1} < \frac{F_2}{b_2}$.

Proposition 5:

(a) If $s_1^\circ = s_2^\circ$, then $\Delta B_1 = \Delta B_2 = 0$.

(b) If $s_1^\circ < s_2^\circ$, then $\Delta B_1 < 0$, $\Delta B_2 > 0$.

Chapter VIII: Mergers and Debt Capacity

Proposition 6:

(a) If $\hat{s}_1 = \hat{s}_2$, then $B(\overline{F}_m) = B(\overline{F}_1) + B(\overline{F}_2)$.

(b) If $\hat{s}_1 \neq \hat{s}_2$, then $B(\overline{F}_m) < B(\overline{F}_1) + B(\overline{F}_2)$.

Corollary 6.1:

A merger between 2 firms leveraged up to their respective debt capacities is never synergistic.

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LIST OF VARIABLES*

Equation	Page	Variable	Definition
	11	s	State obtaining at t = 1, where s [0, $\bar{s}$].
	11	$\bar{s}$	Highest state obtaining at t = 1.
	11	$V(s)$	State-contingent cash flow realized in t = 1.
	11	I	Required investment outlay in t = 1.
	11	a	Technology parameter where V(0) = a.
	11	b	Technology parameter where V'(s) = b.
	11	F	Promised Payment level (t = 1).
	12	$q(s)$	State-pricing (density) function.
(8)	13, 19	$\hat{s}$	Investment threshold state, all-equity firm.
(1)	13	V_o	All-equity firm value (t = 0).
(9)	16, 19	s°	Investment threshold state, levered firm.
(4)	16	B	Value of bonds with promised payment F (t=0).
(5)	17	E	Value of equity claims (t = 0).
(6)	17	V	Levered firm value (t = 0).
(7)	17	A	Agency costs of underinvestment (t = 0).
(20)	21	$\bar{F}$	Promised payment level which maximizes debt value.
(22)	22	$B(\bar{F})$	The firm's debt capacity (maximum debt value).
(33)	23, 33	D	Debt ratio (ratio of debt to firm value).
(28)	27	s°_m	Investment threshold state, merged firm.
(31)	30	ΔA_1	Merger-induced increase in agency costs, firm 1.
(32)	31	ΔA_2	Merger-induced increase in agency costs, firm 2.
(40)	37	U	Debt capacity utilization ratio, ratio of debt value to debt capacity.

*In order of their appearance in paper.

LIST OF VARIABLES (continued)

Equation	Page	Variable	Definition
(44)	39	V_m	Merged firm value.
(47)	47	ΔV	Merger-induced increase in firm value.
(59)	49	B_m	Aggregate merged firm bond value.
(59)	49	ΔB	Merger-induced increase in bond value.
(60)	49	E_m	Aggregate merged firm equity value.
(60)	49	ΔE	Merger-induced increase in equity value.
(72)	59	B_1^m	Value of F_1, post-merger.
(72)	59	ΔB_1	Merger-induced increase in value of F_1.
(73)	59	B_2^m	Value of F_2, post-merger.
(73)	59	ΔB_2	Merger-induced increase in value of F_2.
(78)	63	Δe_1	Merger-induced increase in residual cash flow, project 1.
(79)	64	Δe_2	Merger-induced increase in residual cash flow, project 2.
(80)	67	$\|\Delta e_2\|$	"Coinsurance amount" (value of coinsurance effect).
	73	N_i	Number of outstanding equity shares, unmerged firm i (i = 1, 2).
	74	ΔN_2	Number of additional shares issued by acquiring firm.
(82)	74	P_1	(Cum-dividend) share price of (unmerged) firm 1.
(83)	74	P_2	(Cum-dividend) share price of (unmerged) firm 2.
(84)	74	P_2^*	(Cum-dividend) share price of (merged) firm 2.
(87)	74	G_1	Synergy gain accruing to firm 1.
(87)	74	G_2	Synergy gain accruing to firm 2.
(94)	76	X	Exchange ratio of firm-2 shares for firm-1 shares.

LIST OF VARIABLES (continued)

Equation	Page	Variable	Definition
(95)	77	P_1^*	(Cum-dividend) share price of (about-to-be merged) firm 1.
(104)	80	$\underline{P}_2^*$	Minimum value of merged-firm share price P_2^*.
(105)	80	$\overline{P}_2^*$	Maximum value of merged-firm share price P_2^*.
(112)	84	$\Delta\underline{N}_2$	Minimum number of newly issued shares ΔN_2.
(113)	84	$\Delta\overline{N}_2$	Maximum number of newly issued shares ΔN_2.
(116)	85	$\underline{P}_1^*$	Minimum value of share price P_1^*.
(117)	85	$\overline{P}_1^*$	Maximum value of share price P_1^*.
(118)	85	$\underline{X}$	Minimum value of exchange ratio X.
(119)	85	$\overline{X}$	Maximum value of exchange ratio X.
(126)	90	V_1^m	Cum-dividend equity value accruing to shareholders of firm 1.
(127)	90	V_2^m	Cum-dividend equity value accruing to shareholders of firm 2.
(138)	101	$\overline{V}_1^m$	Nash bargaining solution, firm-1 equity holders.
(139)	101	$\overline{V}_2^m$	Nash bargaining solution, firm-2 equity holders.
(150)	107	$\overline{\overline{V}}_1^m$	Shapley value, firm-1 equity holders.
(151)	107	$\overline{\overline{V}}_2^m$	Shapley value, firm-2 equity holders.
(155)	116	ΔV°	Merger synergy gain in ex-post merger case.
(157)	116	B_1°	Firm-1 bond value in ex-post merger case.
(158)	117	B_2°	Firm-2 bond value in ex-post merger case.
(159)	118	$V_1^{m\circ}$	Firm-1 cum-dividend equity value in ex-post merger case.
(160)	119	$V_2^{m\circ}$	Firm-2 cum-dividend equity value in ex-post merger case.
(163)	120	$\overline{F}_m$	Promised payment level which maximizes merged firm debt value.

LIST OF VARIABLES (continued)

Equation	Page	Variables	Definition
	121	s_m	State for which the merged firm's cash flows just equal the combined investment level.
	129	K	Riskless asset held at t=0 and t=1.
(A.1)	130	V_o^+	All-equity value of firm with existing asset K.
	132	s^+	Investment threshold state, levered firm with existing asset K.
(A.6)	134	B^+	Value of bonds, levered firm with existing asset K.
(A.7)	134	E^+	Value of equity, levered firm with existing asset K.
(A.8)	134	V^+	Levered firm value for firm with existing asset K.
(A.9)	135	A^+	Agency costs of underinvestment for levered firm with existing asset K.
(A.10)	135	ΔA^+	Agency cost savings achieved by introduction of seizable assets K.
(B.3)	138	s^*	Investment threshold state, investment 1, within merged firm.

CHAPTER I

INTRODUCTION

There has been considerable academic and practitioner interest in corporate mergers in recent years. Market-based merger studies, though not unanimous, indicate that bidding shareholders experience non-negative abnormal returns, target shareholders experience large positive abnormal returns, and (bidding and target) debt holders experience non-negative abnormal returns. Thus, the preponderance of this empirical evidence suggests that takeovers increase firm value; however, the source of these merger gains is a largely unsettled question.

Most of the theoretical literature explores the effects of and potential gains to conglomerate merger. Since there is no discernible economic relationship between parties to conglomerate merger, "real" synergies (economies of scale in production, research, distribution and management) are presumably not relevant. Instead, the theoretical studies identify and analyze purely financial effects of mergers (such as the potential for wealth transfers among claimants, or the potential gains from tax or bankruptcy cost savings).

This paper explores the effects of mergers on the investment incentives of the levered firm and on levered firm value. The first part of the paper (Chapters III and IV) consists of an agency-theoretic model of the pre- and post-merger investment policies of two firms with risky debt outstanding. The model of the firm developed highlights the potential for corporate mergers to "improve" investment incentives and thereby bring about a reduction in the agency costs of underinvestment associated with risky debt. Under a fairly broad set of assumptions, it is shown that most firm combinations result in a reduction in agency costs and a concomitant increase in merged firm value. The increment in merged firm value can be considered a sort of "financial" synergy insofar as it depends on capital structure and investment opportunity characteristics rather than technological or operating improvements. Since such "real" synergies are commonly believed absent in today's (predominantly conglomerate) merger market, the idea of "financial" synergies is an appealing explanation for documented conglomerate merger gains.

The model allows the derivation of the debt-equity profiles and investment set characteristics of acquiring and target firms which lead to synergistic mergers (Chapter IV). The potential for mergers to create wealth transfers among claim holders and the relationship of these wealth changes to the merging firms' debt and investment parameters is explored in Chapter V.

The model also yields a notion of debt capacity which is natural in the agency-theoretic setting. The model allows an explicit computation of debt capacity (of the individual firms before merger and the combined firm after merger) in terms of parameters of the underlying investment opportunity sets of the firms. The changes in debt capacity due to merger is characterized in Chapter VIII.

Independent of any modeling considerations, any merger which creates value necessarily involves an apportionment of the gains between merging parties. Chapter VI contains a game-theoretic approach to the problem of apportioning merger gains in a rational and mutually agreeable fashion. Empirical implications of the model are developed for future research and comparison with existing research findings in Chapter IX.

CHAPTER II

RELATED LITERATURE

A growing body of accounting and finance literature is devoted to the investigation and analysis of various aspects of corporate mergers. Many empirical studies investigate information and wealth effects of mergers by examining the stock price behavior of bidding and target firms[1]. While this empirical research has led to a general agreement[2] that target shareholders gain and bidding shareholders (at worst) do not lose, many of thse studies are unable (or do not attempt) to identify the source of the merger gains. The possible sources of merger gains advanced in these and other papers include operating synergies, elimination of inefficient management, creation of monopoly power, exploitation of insider information, and various financial motivations (such as diversification, tax benefits, reduction in the probability of bankruptcy and bankruptcy costs).

Much of the theoretical literature has focused on financial motivations for and effects of mergers based on the assumption that the surge in conglomerate mergers in the 1960's did not involve "technological" or "operating" synergies. In a world without corporate taxes or bankruptcy costs, Myers (1968) and Levy and Sarnat (1970) argue that the diversification effects of merger will not increase the merged firm's value. That is, the merged firm must be worth the simple sum of unmerged firm values since investors can achieve the same level of diversification on personal account.

The Coinsurance Effect. If at least one of the merging firms has risky debt in its capital structure, there is a possibility for reduction in default risk via merger. Lewellan (1971) proposes that (otherwise nonsynergistic) mergers can create value through a reduction of default risk. Lewellan compares the probability that one or both firms default pre-merger to the (joint) probability that both fail post-merger. He shows that the necessary and sufficient condition for the latter to be less than the former is that there be at least one opportunity for one of the merging partners to satisfy a deficiency in the other's ability to meet its debt obligation[3]. Lewellan argues that (merger-induced) default risk reductions would enable the merged firm to borrow more than the unmerged firms could; increased leverage would, in turn, lead to increased merged firm value in a world of corporate taxation and tax-deductible interest payments.

The "coinsurance effect" introduced by Lewellan has been broadened to take into account the possibility of increased payments to bondholders even in the event of default. Since each of the merging firms is made liable for the risky debt of the other, it is easy to prove that post-merger debt payments must equal or exceed the sum of pre-merger debt payments[4]. Higgins and Schall (1975) show that the necessary and sufficient condition for post-merger debt payments to exceed the sum of pre-merger debt payments is that there be at least one case in which a default by one of the merging partners coincides with a positive net equity position by the other. (Clearly, this condition is weaker than Lewellan's requirement for default risk reduction insofar as the solvent firm need not be capable of "bailing out" the insolvent one.) Of course, any post-merger increases in debt payments correspond to decreases in cash available for equity holders. Hence, the coinsurance effect results in a wealth transfer from equity holders to debt holders. Strategies which eliminate this wealth transfer include: (1) retiring all outstanding debt at pre-merger prices and re-issuing new debt after the merger (Higgins and Schall (1975)), (2) issuing additional debt during or following the merger in order to increase the risk (and depress the value) of the pre-merger debt claims (Galai and Masulis (1976)), and (3) using cash and cash equivalents to consummate the merger in order to reduce the amount and "safety" of the assets that can be seized by debt holders in the event of post-merger financial distress (Eger (1983))[5].

Tests of Coinsurance Effect. In several studies, both bondholder and stockholder returns are observed in order to assess whether one set of claimants appears to be gaining at the expense of the other and to gauge the overall impact of mergers. Kim and McConnell (1977) and Asquith and Kim (1982) find that neither acquiring- nor acquired-firm bondholders experience significantly nonzero returns in response to conglomerate mergers. Kim and McConnell also find evidence of increased use of financial leverage by merged companies. Their results are thus consistent with the existence of a coinsurance effect for which the potential wealth transfer to bondholders has been eliminated through increased use of debt financing. Similarly, Asquith and Kim (1982) find that any potential merger-induced wealth transfers among claimants have been neutralized since their (essentially) zero bondholder abnormal returns are accompanied by positive abnormal returns for target shareholders and (essentially) zero abnormal returns for bidding shareholders.

Eger (1983), on the other hand, finds evidence that bidding firm bondholders experience significantly positive abnormal returns around the merger announcement date. The size of the excess bondholder returns found by Eger is similar to that reported by Asquith and Kim (1982); however, unlike Asquith and Kim, Eger's bondholder results are also statistically significant[6]. Like Asquith and Kim, bidding shareholders earn (essentially) zero excess returns and target shareholders earn significantly positive excess returns in the Eger study. Eger's bidding firm results are thus consistent with a wealth transfer from shareholders to bondholders accompanied by synergistic gains accruing to shareholders (and possibly bondholders). Unfortunately, one can neither rule out nor confirm the potential for mergers to create wealth transfers among claimants based on the three cited studies since alternative explanations exist for the findings of each. At best, one can conclude that empirical results are consistent with wealth transfers which have been counterbalanced in the case of the Kim and McConnell and Asquith and Kim studies, and accompanied by synergistic gains in the case of the Eger study.

Debt Capacity. The theoretical analysis of the effects of mergers on various claim values and overall firm value often leads to a consideration of the effect of mergers on debt capacity. Unfortunately, both in the context of mergers and as a separate research issue, debt capacity has been defined in different ways by different authors.

In a model with bankruptcy costs and tax shields, Scott (1977) defined debt capacity as the optimal (i.e., firm-value-maximizing) debt level. In a worked example, Scott demonstrates that a profitable merger need not increase debt capacity (so defined). Scott's definition is unsatisfactory insofar as it eliminates a useful distinction between the amount of debt which is feasible and the amount which maximizes firm value. As Kim (1978) points out, since the optimal capital structure may be unattainable (and therefore irrelevant), a determination of the firm's debt capacity logically precedes the question of optimal capital structure.

Lewellan (1971) defines debt capacity as the amount of debt which can be issued by a firm such that the probability of bankruptcy does not exceed a specified level. Lewellan argues that mergers generally increase debt capacity since they usually bring about a reduction in default risk on the combined outstanding debt. There are two weaknesses in this definition of debt capacity: (1) it blurs the distinction between the impact of mergers on the value of outstanding debt versus capacity to issue additional debt[7], and (2) it does not take into account the fact that debt holders consider all future payments (including those made in the event of default) in valuing the debt.

Stapleton (1982) defines debt capacity as the amount of debt which can be raised by a firm at a given (nominal) yield. This definition is designed to consider the amount of the debt that can be raised when the yield is kept within a "reasonable" range of the prime (or risk-free) rate. While this definition avoids the pitfalls enumerated above, it is unsatisfactory because any choice of range other than that (implicitly) set by the following condition is arbitrary: it is economically plausible that firms continue to borrow (at increasing yields) up to the point where an additional dollar promised does not increase the amount of debt raised. If the foregoing condition is used to redefine the "reasonable" range of yields, Stapleton's definition of debt capacity becomes the same as that used by Myers (1977), Kim (1978), Turnbull (1979), and the one that will be used in this paper--namely, debt capacity is the maximum amount of debt the firm can borrow.

Market Imperfections. The effect of various market imperfections on merger profitability has also been explored in the theoretical literature. Scott (1977) has shown that the combined effect of corporate taxation and tax deductibility of interest payments encourages mergers. In his model, the coinsurance effect is no longer a zero-sum transfer from debt holders to equity holders; instead, equity holders gain tax shields on any additional amounts paid debt holders as a result of the merger. Using a different set of corporate taxation assumptions, Higgins and Schall (1975) show that the tax deductibility of interest payments results in increased firm value only if the merged firm issues additional debt following the merger. Elton, Gruber, and Lightstone (1981) show that corporate taxes may encourage equity holders to consummate value-decreasing mergers if such a merger increases the value of their claims. For all three sets of authors, the existence of bankruptcy costs may or may not contribute to merger profitability.

Among other things, this study models the effect of a third market imperfection--information asymmetry--on investment incentives and firm value. Corporate insiders are modeled to have better information than the market (including creditors) about the investment decisions of the firm. In this context, it has been shown (e.g., Myers (1977)) that the firm will have a tendency to underinvest compared to optimal levels. These investment disincentives cause a loss of firm value (i.e., agency costs of underinvestment) which may be reduced through merger. The details of the framework are explored in the next chapter.

CHAPTER III

ANALYTICAL FRAMEWORK AND METHODOLOGY

In the agency-theoretic models of the firm (see Jensen and Meckling (1976)), the focus is on conflicts between competing claimants to corporate assets. In large part, these conflicts occur because monitoring costs and widely dispersed securities induce outsiders to observe only incompletely private actions by corporate insiders (e.g., investment levels). In this setting, the firm can no longer be viewed as one homogeneous unit whose clear objective is to maximize its market value. In fact, the firm has been portrayed as a collection of players involved in a non-cooperative game (see, e.g., Jensen and Meckling (1976)). When the investment decisions are privately controlled by corporate insiders (who act on the behalf of stockholders), it is easy to show that they have incentives to underinvest relative to total-value-maximizing levels (e.g., Myers (1977)). When this is rationally valued in the price of the external claims, the stockholders eventually bear the costs of deviating from total-value-maximizing investment policies. Such costs have been called agency costs.

The model which follows will allow an explicit computation of post-merger investment policy and the resulting change in agency costs (and equivalently, change in total value). The framework captures two essential features which give rise to agency costs of underinvestment: (1) discretionary investment on the part of the corporate insiders, and (2) an agency relationship between creditors and insiders (in general, external claimants and insiders who control investment choices)[8]. The model is based on technologies which are linear in state and complete markets pricing for financial claims. The salient features of the economy and the valuation rule for pricing the different claims are given below in the context of a representative firm.

(A1) The economy extends through two dates, $t = 0$ and $t = 1$.

(A2) The insiders of each firm control the investment, financing and dividend decisions of the firm and act in the interests of the current shareholders. (For simplicity, no distinction is made between inside equity and outside equity. The only external claims outstanding are risky bonds, as detailed in (A5).)

(A3) The uncertainty in the economy is represented by an Arrow-Debreu state preference model, where $s \in [0, \bar{s}]$ indexes the eventual states of the world to be realized at $t = 1$.

(A4) The firms in the economy are characterized by $\{V(s), I\}$, denoting the investment opportunities available to them at $t = 1$. These technologies are the only asset the firm possesses and they have the following interpretation. The insiders of the firm observe the realized state s at $t = 1$ and choose to invest either zero or to raise (from equity holders) and invest I in the technology. If zero is invested, the investment opportunity lapses and the firm is worthless; if I is invested, $V(s)$ is realized as the resulting cash flow. For convenience, let the states be ordered such that $V(s)$ is monotonically increasing in s. For simplicity, let $V(s)$ be linear and represent it as $V(s) = a + b(s)$ where $a \in R$, $b \in R^{+}$.

(A5) The insiders issue a pure discount bond with a payment F promised at maturity at $t = 1$. The usual priority rule applies such that $\min\{F, C(s)\}$ is pledged the bondholders, where $C(s)$ is the total cash flows resulting from the investment decision. (Here, $C(s) = V(s)$ if investment I is made; otherwise, $C(s) = 0$ for all $s \in [0, \bar{s}]$.)

(A6) A crucial feature of the model is that the external claimants cannot write forcing contracts on the investment decision of the insiders. A sufficient condition for this to be true is that the outsiders would find it prohibitively costly to verify the realized state $s \in [0, \bar{s}]$ such that mutually enforceable state-contingent contracts cannot be written or enforced. Once the debt is issued, the insiders take the investment decisions which maximize the value of stock.

(A7) The pricing of claims will be done, as is customary, by taking expectations of the random cash flow with respect to the unique state price density function, $q(s)$. The current market price of a claim resulting in cash flow $C(s)$ at $t = 1$ in state s would command a current market price of

$$p = \int_0^{\bar{s}} C(s)\, q(s)\, ds$$

Given complete markets, the above pricing rule is valid for either a risk averse or risk neutral economy.

The riskless interest rate is assumed to be zero for simplicity so that

$$\int_0^{\bar{s}} q(s)\, ds = 1/(1 + r_f) = 1$$

Thus, $q(s)$ contains the two essential features of any density function: (1) $q(s) \geq 0$ for $s \in [0, \bar{s}]$[9], and (2) the integral of $q(s)$ over the range $s = [0, \bar{s}]$ is equal to one.

Further assume that $q(s)$ is a uniform density function over $[0, \bar{s}]$. This will simplify valuation computations and allow claim values to be easily related to areas in the figures used.

(A8) To focus on the role played by agency costs, assume that all taxes (corporate and personal) are zero and that bankruptcy costs are zero.

The following time line represents the sequence of economic events and decisions described thus far in assumptions (A1) to (A8):

Issue Debt	State Reveals	Investment Decision	Debt Repaid	Liquidating Dividend
t=0			t=1	

The insider's overall objective is to maximize the value accruing to equity holders from the available investment opportunity[10]. If the firm were all-equity financed, their optimal investment rule would be simple: observe the realized state s, and invest I if $V(s) \geq I$; in other words, invest in non-negative net-present-value projects. Define $\hat{s}$ such that $V(\hat{s}) = I$. Then the all-equity investment rule can be equivalently stated as invest I if $s \geq \hat{s}$. The current value of the investment opportunity set is thus

$$(1)\quad V_o = \int_{\hat{s}}^{\bar{s}} [V(s) - I]\, q(s)\, ds$$

This is the total value which can be obtained from the investment opportunity set available to the equity holders. V_o corresponds to the triangle prx in Figure 1. Given the (A7) assumption that q(s) is uniformly distributed between $[0, \bar{s}]$, $q(s) = 1/\bar{s}$ for $s \in [0, \bar{s}]$. Hence V_o is proportional to area prx, with factor of proportion $1/\bar{s}$. The same proportionality will hold between the values of claims and their corresponding areas contained in Figure 1.

The incentives of the insiders (who take investment decisions on behalf of the shareholders) change when the firm has risky debt outstanding (as given in (A5)) of face value F. At t = 0, the insiders

The Firm's Investment Decision

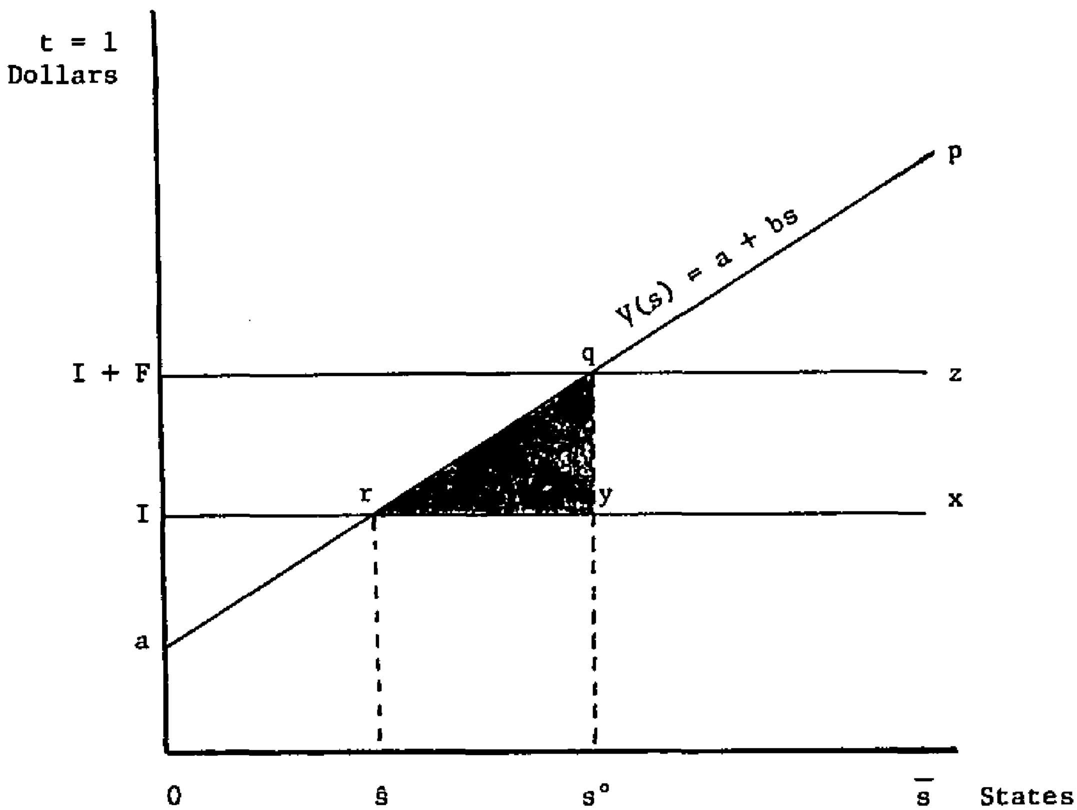

FIGURE 1

have collected a price B from the bondholders, promising to pay the minimum of F or the available cash flow C(s) at t = 1. After observing the realized state s at t = 1, the insiders make the choice between A and B:

(A) Raise (from equity holders) and invest I, obtaining a total cash flow V(s), out of which bondholders must be paid min {F, V(s)}. The residual amount available for the equity holders is V(s) - min {F, V(s)} and the net return of the equity holders is thus

$$-I + V(s) - \min \{F, V(s)\}$$

(B) Invest nothing, obtaining a cash flow of zero such that zero is paid to bondholders and zero is available for equity holders.

The insiders' decision to invest I or zero will be based on which results in higher (net) return to equity holders. Since (at worst) zero is obtainable for equity holders (decision B above), the investment will only be undertaken if

$$-I + V(s) - \min \{F, V(s)\} \geq 0$$

$$V(s) - I + \max \{-F, -V(s)\} \geq 0$$

$$\text{(2)} \quad \text{Max} \{V(s) - I - F, -I\} \geq 0$$

Equation (2) compares the investment payoff (L.H.S.) to the noninvestment payoff (R.H.S.). Further, the investment payoff reflects two possibilities--that of investment and default (with return -I) versus investment and repayment (with return V(s) - I - F). The former strategy dominates the latter (-I > V(s) - I - F) if and only if V(s) - F < 0. (That is, it is better to invest and default than to invest and repay i.f.f. the debt burden F exceeds cash flow V(s).) However, even if this condition holds, the strategy of noninvestment yields an even higher return of zero.

Since the "invest and default" strategy is always dominated by one of noninvestment, the corresponding payoff can be eliminated from equation (2), as follows:

(3) $V(s) - I - F \geq 0$

Now define s° such that $V(s^\circ) = I + F$. Then the investment decision rule implicit in equation (3) becomes: invest in the levered firm for $s \geq s^\circ$.

From the above discussion, it is clear that the bondholders and stockholders get a zero return in noninvestment states $s \in [0, s^\circ]$. Further, the bondholders get the promised payment F and the equity holders get a net payoff $V(s) - I - F$ in investment states $s \in [s^\circ, \bar{s}]$.

Ex post, the insiders are choosing an investment policy which maximizes the value of the claims of stockholders, even though it involves giving up positive net-present-value projects in states $s \in (\hat{s}, s^\circ)$. As long as bondholders are not able to write enforceable contracts preventing this investment behavior (see (A6)), they will price the bonds with the rational expectation that insiders take investment decisions which maximize stockholder wealth. The price B of the bonds that they will be willing to pay at time $t = 0$ is thus only

$$B = \int_0^{\bar{s}} \min \{F, C(s)\}\, q(s)\, ds$$

$$B = \int_0^{s^\circ} \min \{F, 0\}\, q(s)\, ds + \int_{s^\circ}^{\bar{s}} \min \{F, V(s)\}\, q(s)\, ds$$

$$B = \int_0^{s^\circ} 0\, q(s)\, ds + \int_{s^\circ}^{\bar{s}} F\, q(s)\, ds$$

$$(4)\quad B = \int_{s^\circ}^{\bar{s}} F\, q(s)\, ds$$

Thus, bond value B is proportional to the rectangle zqyx in Figure 1. The levered equity claims, with value E, will be priced at time $t = 0$ as

$$E = \int_{0}^{\bar{s}} \text{Max}\ \{C(s) - F,\ 0\}\ q(s)\ ds$$

$$E = \int_{0}^{s^\circ} \text{Max}\ \{-F,\ 0\}\ q(s)\ ds + \int_{s^\circ}^{\bar{s}} [\text{Max}\ \{V(s)-F,\ 0\} - I]\ q(s)\ ds$$

$$E = \int_{0}^{s^\circ} 0\ q(s)\ ds + \int_{s^\circ}^{\bar{s}} [V(s)-I-F]\ q(s)\ ds$$

$$(5)\quad E = \int_{s^\circ}^{\bar{s}} [V(s)-I-F]\ q(s)\ ds$$

Equity value E is proportional to triangle pqz in Figure 1. It is easy to solve for the levered firm value V which accrues to the owners of the investment opportunity by setting $V = B + E$, as follows:

$$V = B + E = \int_{s^\circ}^{\bar{s}} F\ q(s)\ ds + \int_{s^\circ}^{\bar{s}} [V(s)-I-F]\ q(s)\ ds$$

$$(6)\quad V = \int_{s^\circ}^{\bar{s}} [V(s) - I]\ q(s)\ ds$$

Levered firm value V is thus proportional to trapezoid pqyx in Figure 1. By comparing levered- and unlevered-firm values in Figure 1, it is apparent that there is a residual loss in firm value corresponding to shaded triangle qry. More formally, subtract levered firm value V (equation (6)) from the unlevered firm value V_o (equation (1)) to solve for the residual loss in value, denoted A:

$$A = V_o - V = \int_{\hat{s}}^{\bar{s}} [V(s) - I]\ q(s)\ ds - \int_{s^\circ}^{\bar{s}} [V(s) - I]\ q(s)\ ds$$

$$(7)\quad A = \int_{\hat{s}}^{s^\circ} [V(s) - I]\ q(s)\ ds$$

where A is proportional to shaded triangle qry in Figure 1. This loss in value is an agency cost (of risky debt) which results from the incentives of insiders to underinvest in the levered firm.

Summarizing, once the debt has been issued, insiders pursue an investment policy which maximizes t=1 cash flows to equity, even though this deviates form the value-maximizing rule that all positive net-present-value projects be accepted. As a result, levered firm value is lower than that attained by the all-equity firm, which sets a value-maximizing investment policy. With proper pricing of bonds by a debt market which rationally anticipates these investment disincentives, equity holders bear all the agency costs of the distorted investment policy. It is therefore in the owners' interest to devise ways to resolve or ameliorate the problem, such as by undertaking debt covenants or monitoring and auditing activities. For example, if assumption (A6) is relaxed so that state-contingent contracts are enforceable, it would be in the equity holders' interests to precommit to the value-maximizing investment policy (invest I if $s \geq \hat{s}$)[11]. However, as discussed by Myers (1977), it is difficult to envision real-world scenarios in which the problem is costlessly resolved. In this context, it would be interesting to consider whether and under what conditions mergers bring about an improvement in underinvestment incentives.

The agency costs identified above have been derived from a model in which firm value is entirely contingent on future investment outlays. As Myers (1977) points out, the model is not as unrealistic as it may appear at first blush, since the ultimate value of most (noncash) assets depends, to a greater or lesser degree, on some future discretionary investments--e.g., maintenance expenditures, research and development costs, advertising outlays. Moreover, as shown in Appendix A, the model is quite robust insofar as it can be used to demonstrate that the underinvestment problem exists for any firm with risky debt in its capital structure and some investment contingent projects in its asset structure. As discussed in Appendix A, equity holders become more eager to invest when a refusal to do so results in forfeiture of existing assets. The underinvestment problem remains, however, since positive net-present-value projects are still rejected (albeit in fewer states than before). Since the addition of "seizable" assets complicates the analysis without eliminating underinvestment incentives (and related agency costs), simplifying assumption (A4) is retained in the remainder of the paper.

Note that given the state space $[0, \bar{s}]$, all the relevant exogenous characteristics of the firm are given by the technology $V(s) = a + bs$, the required investment I, and the promised payment F of the outstanding debt. Thus, the shorthand representation $\{a, b, I, F\}$ can be used to refer to a particular firm. States $\hat{s}$, s° and the various claim values can be expressed in terms of these given parameters. By definition,

$$V(\hat{s}) = I$$

$$a + b\hat{s} = I$$

$$\hat{s} = [I - a]/b \tag{8}$$

Similarly,

$$V(s^\circ) = I + F$$

$$a + bs^\circ = I + F$$

$$s^\circ = [I + F - a]/b \tag{9}$$

States $\hat{s}$ and s° will play an important role in the analysis. For example, it is clear from equation (9) that s° is increasing in F. Thus, the underinvestment problem is more severe in highly levered firms since the interval of investment states $[s^\circ, \bar{s}]$ contracts as the promised payment level increases.

Performing the integration indicated (see equations (1), (4), (5), and (7)) solves for the values of the all-equity firm, debt, equity, and agency loss, respectively, as follows:

$$(10)\quad V_o = \frac{b}{2\bar{s}}\,[\bar{s} - \hat{s}]^2$$

$$(11)\quad B = \frac{F}{\bar{s}}\,[\bar{s} - s^\circ] = \frac{b}{\bar{s}}\,[\bar{s} - s^\circ][s^\circ - \hat{s}]$$

$$(12)\quad E = \frac{b}{2\bar{s}}\,[\bar{s} - s^\circ]^2$$

$$(13)\quad A = \frac{b}{2\bar{s}}\,[s^\circ - \hat{s}]^2$$

Of course, $\hat{s}$ and s° can be substituted for in terms of the firm parameters via equations (8) and (9). In particular, debt, equity and agency cost values can each be expressed as a function of F, the level of promised payment. The pivotal importance of $\hat{s}$ and s° to the claim values is based on the fact that the firm is worthless when the investment is passed over.

To more thoroughly examine the effect of the promised payment level F on claim values, substitute the equation (9) expression for s° into equations (11) through (13) and simplify as follows:

$$(14)\quad B(F) = \frac{F}{b\bar{s}}\,[V(\bar{s}) - I - F]$$

$$(15)\quad E(F) = \frac{1}{2b\bar{s}}\,[V(\bar{s}) - I - F]^2$$

$$(16)\quad A(F) = \frac{F^2}{2b\bar{s}}$$

As shown in equation (7), firm value V can be expressed as the difference between the all-equity value of the firm V_o and the agency costs A

($V = V_o - A$). Then using the equation (10) expression for V_o and the equation (16) expression for A, V can be written as

$$\text{(17)}\quad V(F) = V_o - A(F) = \frac{b}{2\bar{s}}[\bar{s} - \hat{s}]^2 - \frac{F^2}{2b\bar{s}}$$

By taking first and second derivatives with respect to F in equations (15) through (17) above, it is easy to show that E(F) is monotonically decreasing and convex in F. A(F) is monotonically increasing and convex in F. V(F) is monotonically decreasing and concave in F[12].

As can be seen in equation (14) above, the value of debt is not monotonically increasing in F. In fact, there is a well-defined maximum amount of debt the firm can raise. Define this maximum as the firm's debt capacity, denoted $B(\bar{F})$, where $\bar{F}$ is the level of promised payment which maximizes B(F). In order to solve for the firm's debt capacity, it is useful to re-express B(F), the value of the bonds given in equation (11), as

$$\text{(18)}\quad B(F) = \frac{F}{\bar{s}}[\bar{s} - (I + F - a)/b]$$

$$\text{(19)}\quad B(F) = F - \frac{F^2}{b\bar{s}} - \frac{F(I - a)}{b\bar{s}}$$

From equation (19), it is clear that B(F) is concave in F. In order to solve for $\bar{F}$, the level of promised payment which maximizes B(F):

$$B'(\bar{F}) = 1 - \frac{2\bar{F}}{b\bar{s}} - \frac{(I - a)}{b\bar{s}} = 0$$

$$\text{(20)}\quad \bar{F} = \tfrac{1}{2}[a + b\bar{s} - I]$$

$$\text{(21)}\quad \bar{F} = \tfrac{1}{2}[V(\bar{s}) - I]$$

In order to solve for $B(\bar{F})$, the debt capacity, rewrite equation (18) with $\bar{F}$ replacing F as follows

$$B(\overline{F}) = \frac{\overline{F}}{b\overline{s}} [b\overline{s} - I - \overline{F} + a]$$

$$(22) \quad B(\overline{F}) = \frac{\overline{F}}{b\overline{s}} [V(\overline{s}) - I - \overline{F}]$$

From equation (21), we know $2\overline{F} = V(\overline{s}) - I$. Then equation (22) can be rewritten

$$B(\overline{F}) = \frac{\overline{F}}{b\overline{s}} [2\overline{F} - \overline{F}]$$

$$(23) \quad B(\overline{F}) = \frac{\overline{F}^2}{b\overline{s}}$$

Finally, substituting the equation (20) expression for $\overline{F}$ into equation (23) yields

$$B(\overline{F}) = \frac{1}{4b\overline{s}} [b\overline{s} - (I - a)]^2$$

$$B(\overline{F}) = \frac{1}{4b\overline{s}} [b\overline{s} - b\hat{s}]^2$$

$$(24) \quad B(\overline{F}) = \frac{b}{4\overline{s}} [\overline{s} - \hat{s}]^2$$

$$(25) \quad B(\overline{F}) = \tfrac{1}{2}V_o$$

Starting from the equation (15) and (16) expressions for E(F) and A(F), and using computations which are virtually identical to those made in equations (22) through (24) above, it can easily be shown that

$$(26) \quad E(\overline{F}) = A(\overline{F}) = \tfrac{1}{4}V_o$$

Equation (26) states that at debt capacity, the value of the equity claims is the same as the agency costs of underinvestment, and that both are equal to one-quarter the all-equity value of the firm. Equation (25) states that the debt capacity of the firm is half its all-equity value.

This level of debt capacity is in sharp contrast to that attainable given perfect capital markets, where the entire firm value (which does not vary if F) can be raised as debt. Here, not only is total firm value decreasing in debt, but the maximum amount of debt that can be raised is only two-thirds of the firm value V remaining after agency cost reductions. The latter point can be seen by defining D(F) as the ratio of debt to firm value. Using equations (25) and (26) above, $D(\overline{F})$ is easily expressed as

$$(27)\quad D(\overline{F}) = \frac{B(\overline{F})}{B(\overline{F}) + E(\overline{F})} = \frac{\frac{1}{2}V_o}{\frac{1}{2}V_o + \frac{1}{4}V_o} = 2/3$$

Thus, the maximum amount of debt that can be raised is only two-thirds of the levered firm value V.

The relationships discussed above between agency costs, firm and claim values as a function of F are depicted in Figure 2. Since firm value V is monotonically decreasing if F, the "optimal" (firm-value-maximizing) debt level is zero. This is because any level of promised payment induces the insiders to pass over positive net-present-value projects in some future states. However, when other leverage-related benefits (not explicitly modeled here) are considered, the firm may choose a non-zero "optimal" level of debt. Such leverage-related benefits might include initial capital requirements, interest tax shields, reducing agency costs of equity (Jensen and Meckling (1976)), and the signalling value of debt (Ross (1977)).

Claim Values as a Function of

Promised Payment Level

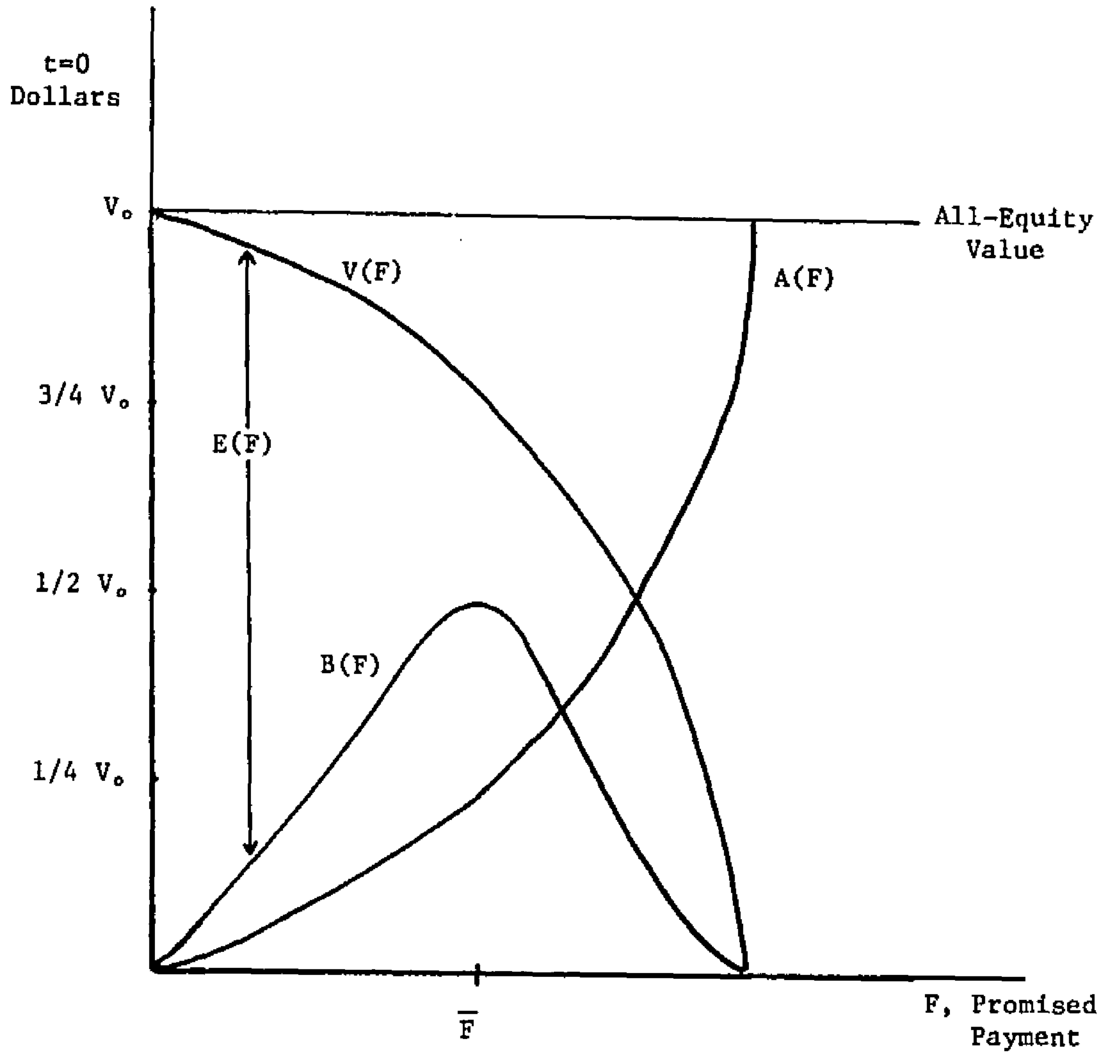

FIGURE 2

CHAPTER IV

FINANCIAL SYNERGY OF MERGERS

This section contains an exploration of how the investment incentives of insiders can change as a result of corporate merger, which leads to a change in the agency costs. A merger which results in a net reduction in agency costs can be described as synergistic or value-increasing since the merged firm's value exceeds the sum of unmerged firm values. Since the synergies achieved are not related to technological or operating improvements (such as the achievement of economies of scale in production) and instead depend on the interaction of the firm's investment opportunities and financing decisions, any value increases brought about by merger are called "financial" synergies. Even though the underlying technologies (investment opportunities available) and aggregate level of outside financing are unaltered by merger, most mergers will cause changes in investment incentives which will, in turn, affect overall firm value. The analysis in this section will indicate the source of financial synergies and the characteristics of merging firms which give rise to synergistic mergers.

Many of the results derived in this section can be given stronger interpretation if the following additional assumption is made:

(A9) In the poorest state, the firm can invest in zero net-present-value projects by using capital markets. An immediate implication of this assumption is $a + b(0) - I = 0$, or $a - I = 0$.

Note that without assumption (A9), the technology of the firm can in general have $a - I < 0$ -- i.e., in states $s < \hat{s}$, the firm has only negative net-present-value projects available. Assumption (A9) focuses on the capital market investments which are available to any capital market agent, including firms, in all states. If these investments are zero net-present-value investments to the firm, then $V(s) - I \geq 0$ for all $s \in [0, \bar{s}]$ and $V(s) - I = 0$ for $s = 0$. Using the equation (8) definition of $\hat{s}$ and equation (9) definition of $s°$, under (A9) $\hat{s} = 0$ and $s° = F/b$. Note that (A9) is a temporary assumption and unless explicitly mentioned, only assumption (A1) to (A8) apply in the discussion.

Consider two firms, firms 1 and 2, with investment opportunities given by $V_1(s) = a_1 + b_1 s$ and $V_2(s) = a_2 + b_2 s$. The firms are characterized with required investment outlays I_1 and I_2, and promised payments F_1 and F_2, respectively. In short, the firms can be represented as $\{a_1, b_1, I_1, F_1\}$ and $\{a_2, b_2, I_2, F_2\}$. The analysis of chapter III can be used to determine $s_1°$ and $s_2°$, the poorest state for which insiders of firms 1 and 2 would choose to invest.

If firms 1 and 2 merge, the new merged firm has access to the combined technology $V_m(s) = a_1 + a_2 + (b_1 + b_2)s$ with a required investment of $(I_1 + I_2)$. Thus, the merged firm can be represented as $\{a_1 + a_2, b_1 + b_2, I_1 + I_2, F_1 + F_2\}$. It is assumed throughout that the merger is accomplished via an exchange of shares in which 100% of target firm equity is acquired (i.e., a 100% pure exchange merger). A cash-financed merger is not possible since the only asset each of the unmerged firms possesses is the option to exercise their respective $t = 1$ investment opportunities[13]. A debt-financed merger would mean the merged firm debt would exceed $F_1 + F_2$, making it difficult to distinguish the incentive effects of additional debt from those of the merger itself. The merged firm's technology, promised payment and required investment is graphically represented in Figure 3. For clarity, the net value of the investment opportunity, $V(s) - I$, is shown for both merged and unmerged firms in the figure.

Recall that insiders of the firm maximize the current value of equity claims by investing I in states for which $V(s) - I \geq F$. Equivalently stated, insiders invest whenever $s \geq s^\circ$. Thus, in Figure 3, firm 1 (with promised payment F_1) invests in states $s \geq s_1^\circ$; firm 2 (with promised payment $F_2 = F_1$) invests in states $s \geq s_2^\circ$. For reasons identical to those set forth for unmerged firms in Chapter III, insiders of the merged firm maximize the current value of their equity holders' claims by investing $I_1 + I_2$ in states for which $V_m(s) - I_1 - I_2 \geq F_1 + F_2$. Define s_m° as the smallest state in which combined values of both investment opportunities just cover the combined promised payment. That is,

(28) $V_m(s_m^\circ) - I_1 - I_2 = F_1 + F_2$

Then the merged firm will undertake both investments for states $s \geq s_m^\circ$; otherwise, neither investment will be undertaken.

The possibility that the merged firm would optimally exercise only one of its investment options while allowing the other to lapse is explored in Appendix B. It will be assumed throughout the remainder of the paper that it is appropriate for the merged firm to pursue a "both-or-neither-project" investment policy under the fairly broad necessary and sufficient conditions specified in Appendix B (namely, reasonably high combined debt levels and/or similar profitability levels for the two investments).

The Merged Firm's Investment Decision

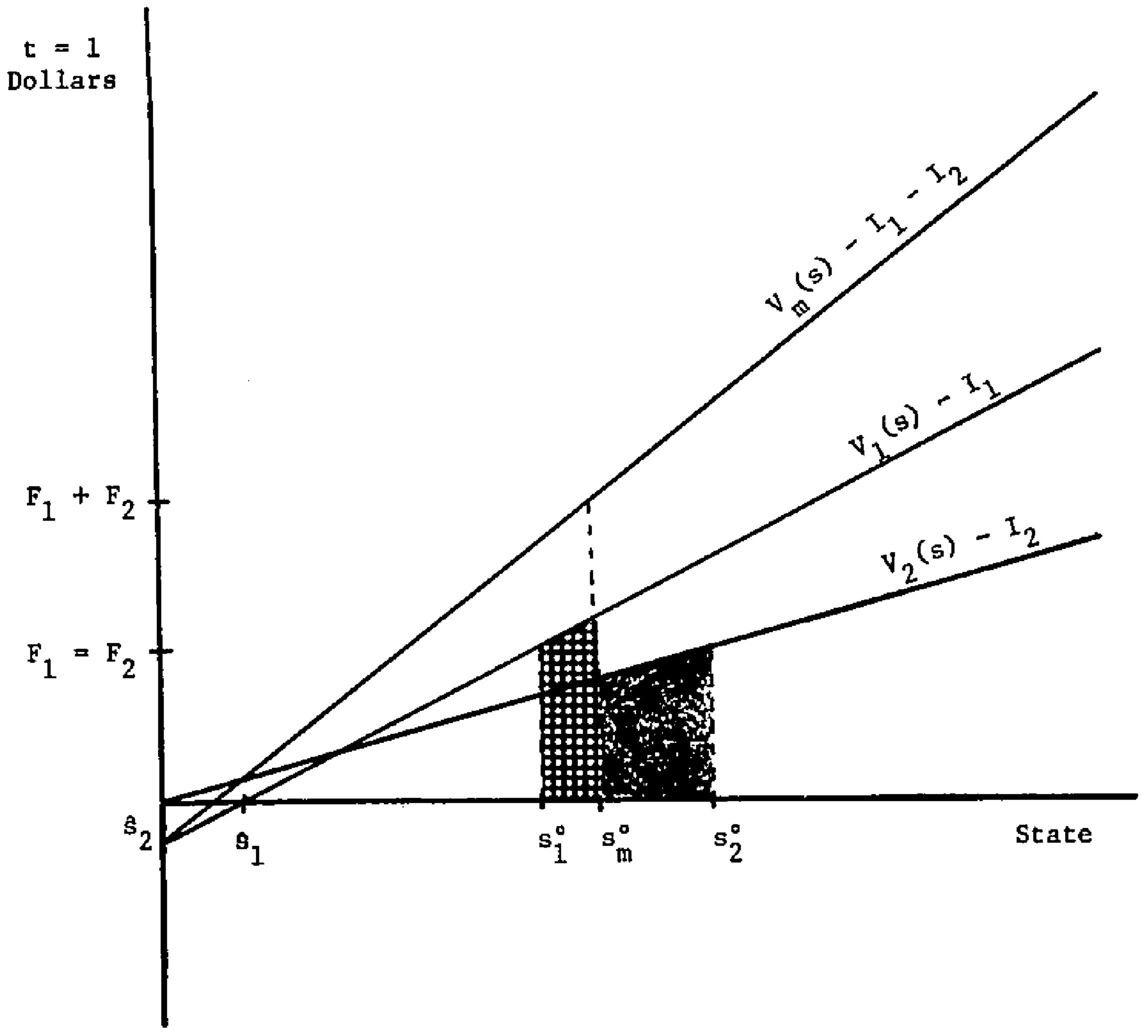

FIGURE 3

From equation (28), we can solve for s_m° as

$$(29)\quad s_m^\circ = \frac{I_1 + I_2 + F_1 + F_2 - a_1 - a_2}{b_1 + b_2}$$

It is clear from equation (29) that s_m° is related to s_1° and s_2°. This relationship is characterized below in Proposition I.

Proposition I:

(a) $s_1^\circ = s_2^\circ$ if and only if $s_m^\circ = s_1^\circ = s_2^\circ$.

(b) $s_1^\circ \neq s_2^\circ$ (w.l.o.g. $s_1^\circ < s_2^\circ$) if and only if $s_1^\circ < s_m^\circ < s_2^\circ$.

Proof: The proof follows easily from the definitions of s_1°, s_2° and s_m°. From equation (9),

$$s_1^\circ = \frac{I_1 + F_1 - a_1}{b_1} \qquad\qquad s_2^\circ = \frac{I_2 + F_2 - a_2}{b_2}$$

Using the equation (29) definition of s_m°,

$$s_m^\circ = \frac{I_1 + F_1 - a_1}{b_1 + b_2} + \frac{I_2 + F_2 - a_2}{b_1 + b_2}$$

$$(30)\quad s_m^\circ = \frac{b_1}{b_1 + b_2}(s_1^\circ) + \frac{b_2}{b_1 + b_2}(s_2^\circ)$$

where b_1, $b_2 > 0$. Then s_m° is a strictly convex combination of s_1° and s_2°, in which case it is clear $s_m^\circ = s_1^\circ = s_2^\circ$ i.f.f. $s_1^\circ = s_2^\circ$. Otherwise ($s_1^\circ \neq s_2^\circ$), it must be $s_1^\circ < s_m^\circ < s_2^\circ$. The reverse implication is immediate. Q.E.D.

Proposition I says that if before merger, the firms would have utilized their technologies in identical states (i.e., $s_1^\circ = s_2^\circ$), then the utilization of the technologies is not changed by merger. The merged firm will exercise both the options in the same set of states that the unmerged firms do. In this case, there is no change in the investment incentives or agency costs as a result of the merger.

However, it is more likely that the pre-merger utilization of the technology differs. Without loss of generality, designate the firm with larger s° as firm 2--i.e., if $s_1^\circ \neq s_2^\circ$, assume $s_1^\circ < s_2^\circ$. The proposition states that if the pre-merger utilizations of technologies differ, s_m° is between s_1° and s_2° (w.l.o.g., $s_1^\circ < s_m^\circ < s_2^\circ$). This means the merged firm exercises the firm-1 investment in fewer states than firm 1 does standing alone. Specifically, in states $s \in [s_1^\circ, s_m^\circ)$, the merged firm chooses to pass over the investment opportunity that would have been exercised by firm 1 alone. The merger thus induces an increase in the agency costs attributable to the firm-1 investment. The increase in agency costs, denoted ΔA_1, can be computed by subtracting the pre-merger agency costs (equation (7)) from its post-merger counterpart, as follows:

$$\Delta A_1 = \int_{\hat{s}_1}^{s_m^\circ} [V_1(s) - I_1]\, q(s)\, ds - A_1$$

$$= \int_{\hat{s}_1}^{s_m^\circ} [V_1(s) - I_1]\, q(s)\, ds - \int_{\hat{s}_1}^{s_1^\circ} [V_1(s) - I_1]\, q(s)\, ds$$

$$(31)\quad \Delta A_1 = \int_{s_1^\circ}^{s_m^\circ} [V_1(s) - I_1]\, q(s)\, ds$$

The post-merger increase in agency costs for the firm-1 investment is represented by the cross-hatched area in Figure 3.

On the other hand, the merged firm exercises the firm-2 investment in more states th.~ firm 2 does standing alone. In states $s \in [s_m^\circ, s_2^\circ)$, the merged firm exercises the investment that would have been passed over by an unmerged firm 2. The merger induces a decrease in the agency costs attributable to underinvestment in the firm-2 investment.

The (negative) increase in agency costs, denoted ΔA_2, can be computed by subtracting the pre-merger agency costs (equation (7)) from its post-merger counterpart, as follows:

$$\Delta A_2 = \int_{\hat{s}_2}^{s_m^\circ} [V_2(s) - I_2]\ q(s)\ ds - A_2$$

$$= \int_{\hat{s}_2}^{s_m^\circ} [V_2(s) - I_2]\ q(s)\ ds - \int_{\hat{s}_2}^{s_2^\circ} [V_2(s) - I_2]\ q(s)\ ds$$

$$(32)\quad \Delta A_2 = \int_{s_2^\circ}^{s_m^\circ} [V_2(s) - I_2]\ q(s)\ ds = - \int_{s_m^\circ}^{s_2^\circ} [V_2(s) - I_2]\ q(s)\ ds$$

The post-merger decrease in agency costs of underinvestment for firm 2 corresponds to the shaded area in Figure 3.

The merged firm value is greater than the sum of unmerged firm values if the combination of the agency cost increases is negative, indicating a net decrease in agency costs ($\Delta A_1 + \Delta A_2 < 0$). On the other hand, if the sum of the agency cost increases is positive, agency costs are greater in the merged firm than they were for component firms ($\Delta A_1 + \Delta A_2 > 0$). Value-increasing mergers are often described as being "synergistic" in the research literature. Here, any merger-induced value increases result from (a type of) financial synergy equal to net agency cost savings. The merger represented in Figure 3 is synergistic since decreases in the agency costs for firm 2 exceed increases in the agency costs for firm 1.

Corollary 1.1 below follows from Proposition 1 and establishes a relationship between the ordering of the s°'s and the sign of the agency cost increase for component investments.

<u>Corollary 1.1:</u>

(a) $s_1^\circ = s_2^\circ$ if and only if $\Delta A_1 = \Delta A_2 = 0$.

(b) $s_1^\circ < s_2^\circ$ if and only if $\Delta A_1 > 0$, $\Delta A_2 < 0$.

Proof: From equations (31) and (32),

$$\Delta A_1 = \int_{s_1^\circ}^{s_m^\circ}[V_1(s) - I_1]\, q(s)\, ds \qquad \Delta A_2 = \int_{s_2^\circ}^{s_m^\circ}[V_2(s) - I_2]\, q(s)\, ds$$

(a) From Proposition I, if $s_1^\circ = s_2^\circ$, then $s_m^\circ = s_1^\circ = s_2^\circ$, in which case each of the above integrals for ΔA_1 and ΔA_2 must equal zero.

For the reverse implication, given $\Delta A_1 = 0$ and $\Delta A_2 = 0$, it will be shown that it must be $s_1^\circ = s_2^\circ$ since $s_1^\circ \neq s_2^\circ$ involves a contradiction. Suppose $s_1^\circ \neq s_2^\circ$. Then Proposition I implies $s_1^\circ < s_m^\circ < s_2^\circ$. Since $\hat{s}_1 < s_1^\circ$ (by definition), $V_1(s) - I_1 > 0$ for all $s \in [s_1^\circ, s_m^\circ]$. This implies $\Delta A_1 > 0$, which contradicts $\Delta A_1 = 0$.

(b) If $s_1^\circ < s_2^\circ$, it must be $s_1^\circ < s_m^\circ < s_2^\circ$ from Proposition I. Since $\hat{s}_1 < s_1^\circ$ by definition, $V_1(s) - I_1 > 0$ for all $s \in [s_1^\circ, s_m^\circ]$, implying $\Delta A_1 > 0$. Similarly, $\hat{s}_2 < s_m^\circ$ (see Appendix B) and $s_m^\circ < s_2^\circ$ (Proposition I) implies $V_2(s) - I_2 > 0$ for $s \in [s_m^\circ, s_2^\circ]$. Therefore, $\Delta A_2 < 0$.

For the reverse implication, given $\Delta A_1 > 0$ and $\Delta A_2 < 0$, it will be shown that $s_1^\circ < s_2^\circ$. $\Delta A_1 > 0$ implies $s_m^\circ > s_1^\circ$ since $[V_1(s) - I_1]\, q(s)$ is positive for $s > s_1^\circ$. From Proposition I, $s_m^\circ > s_1^\circ$ implies $s_1^\circ \neq s_2^\circ$ and therefore $s_1^\circ < s_m^\circ < s_2^\circ$ (equivalently, $s_1^\circ < s_2^\circ$). Q.E.D.

The corollary states that whenever the pre-merger utilization of technologies is identical for firms 1 and 2, there will be no change in the agency costs of either firm as a result of the merger. In this case, there is no possibility for a synergistic merger, and therefore no motive for the two firms to merge.

If, on the other hand, $s_1^\circ \neq s_2^\circ$ (and therefore $s_1^\circ < s_2^\circ$), each firm will experience a change in agency costs as a result of merger; further, it cannot be the case that both firms experience an increase in agency costs (or both experience a decrease). Instead, there will always be one firm which experiences a decrease in agency costs (firm 2) while the other experiences an increase (firm 1).

In other words, for any two firms for which $s_1^\circ \neq s_2^\circ$, the firm with larger s° (firm 2) stands to gain by merger, whereas the other firm (firm 1) stands to lose. It can thus be argued that only firm 2 has a motive to initiate a merger or is in a position to pay a premium to bring about a merger. Firm 1, on the other hand, has no motive to initiate a merger nor could it afford to pay a premium to bring one about. In fact, firm 1 must be paid a premium (or side payment) in order to "break even" or gain in a merger.

It would be useful to characterize s° in terms of some easily understood ratios. In the following discussion, s° is related to a debt ratio D and a debt capacity utilization ratio U. Each of these ratios not only takes into account the debt level (promised payment F), but also the productivity of the firm's technology.

As was done in Chapter III, define the ratio of debt to firm value as

$$(33)\quad D = \frac{B(F)}{B(F) + E(F)}$$

where B(F) and E(F) are the values of debt and equity (respectively) in the levered firm when F is the promised payment on outstanding debt. Using the definitions for B(F) and E(F) provided in equations (11) and (12), D can be expressed as

$$D = \frac{\frac{b}{\bar{s}}(\bar{s} - s^\circ)(s^\circ - \hat{s})}{\frac{b}{\bar{s}}(\bar{s} - s^\circ)(s^\circ - \hat{s}) + \frac{b}{2\bar{s}}(\bar{s} - s^\circ)^2}$$

$$(34)\quad D = \frac{s^\circ - \hat{s}}{(s^\circ - \hat{s}) + \frac{1}{2}(\bar{s} - s^\circ)}$$

$$(35)\quad D = \frac{s^\circ - \hat{s}}{\frac{1}{2}(\bar{s} + s^\circ) - \hat{s}}$$

We can determine whether D is increasing or decreasing in s° by determining the sign of

$$(36)\quad \partial D/\partial s° = \frac{[\tfrac{1}{2}(\bar{s} + s°) - \hat{s}] - \tfrac{1}{2}(s° - \hat{s})}{[\tfrac{1}{2}(\bar{s} + s°) - \hat{s}]^2}$$

Since the denominator of $\partial D/\partial s°$ is a squared term, the sign of $\partial D/\partial s°$ is determined by the sign of the numerator. The numerator in equation (36) simplifies to $\tfrac{1}{2}(\bar{s} - \hat{s})$, where $\tfrac{1}{2}(\bar{s} - \hat{s}) > 0$ (since $\hat{s} < \bar{s}$ always). Therefore, $\partial D/\partial s°$ is increasing in s°:

$$(37)\quad \partial D/\partial s° > 0$$

Based on the above, s° can be related to D by the following corollary,

<u>Corollary 2.1:</u>

Given $\hat{s}_1 = \hat{s}_2$,

(a) $s_1° = s_2°$ if and only if $D_1 = D_2$.

(b) $s_1° < s_2°$ if and only if $D_1 < D_2$.

<u>Proof:</u> From equation (35), $D(\hat{s}, s°) \triangleq \dfrac{s° - \hat{s}}{\tfrac{1}{2}(\bar{s} + s°) - \hat{s}}$. Fix the value of $\hat{s} = \hat{s}_1 = \hat{s}_2$. From equation (37), it is clear that $D(\hat{s}, \cdot)$ is a strictly increasing function of s°. Therefore, $s_1° = s_2°$ if and only if $D(\hat{s}, s_1°) = D(\hat{s}, s_2°)$. Similarly, $s_1° < s_2°$ if and only if $D(\hat{s}, s_1°) < D(\hat{s}, s_2°)$. Q.E.D.

For firms with identical[14] $\hat{s}$'s the corollary establishes the firm's debt ratio D as a real-world referent for s°. To emphasize the extent to which s° and D can be identified with one another, it is interesting to see that s° and D react in a similar fashion to technology parameters a, b, I and financial variable F. That is, the partial derivatives of D and s° with respect to a, b, I and F have the same sign (respectively). From equation (34),

$$D = \frac{s^\circ - \hat{s}}{(s^\circ - \hat{s}) + \frac{1}{2}(\bar{s} - s^\circ)}$$

From equation (11), it is clear that $F = b(s^\circ - \hat{s})$ or $F/b = s^\circ - \hat{s}$. Using the equation (9) expression for s°,

$$\bar{s} - s^\circ = \bar{s} - \frac{(I + F - a)}{b} = \frac{1}{b}[b\bar{s} - (I + F - a)]$$

Substituting the above expressions for $(s^\circ - \hat{s})$ and $(\bar{s} - s^\circ)$ into equation (34) yields

$$D = \frac{\frac{F}{b}}{\frac{F}{b} + \frac{1}{2b}[b\bar{s} - (I + F - a)]}$$

(38) $$D = \frac{F}{\frac{1}{2}F + \frac{1}{2}(b\bar{s} - I + a)}$$

From equation (38), it can easily be shown that

$$\partial D/\partial F > 0 \qquad \partial D/\partial I > 0 \qquad \partial D/\partial b < 0 \qquad \partial D/\partial a < 0$$

From equation (9), $s^\circ = [I + F - a]/b$. Therefore,

$$\partial s^\circ/\partial F > 0 \qquad \partial s^\circ/\partial I > 0 \qquad \partial s^\circ/\partial b < 0 \qquad \partial s^\circ/\partial a < 0$$

Thus, D and s° respond to changes in a, b, I and F in a similar fashion. Equity holders are more reluctant to invest (i.e., s° increases) as leverage and required investment levels rise. They are more eager to invest (s° decreases) as profitability rises (a, b). The leverage ratio D can serve as a surrogate for s° since it reflects the fact that firm value is lower (and hence D higher) for a firm with poor investment incentives (s° high) than in a firm with good investment incentives (s° low).

In Corollary 1.3, a direct link is established between the debt ratio and the merger-induced increases in agency costs, ΔA_1 and ΔA_2.

Corollary 1.3:

Given $\hat{s}_1 = \hat{s}_2$,

(a) $D_1 = D_2$ if and only if $\Delta A_1 = \Delta A_2 = 0$.

(b) $D_1 < D_2$ if and only if $\Delta A_1 > 0$, $\Delta A_2 < 0$.

Proof: Given $\hat{s}_1 = \hat{s}_2$,

(a) $D_1 = D_2$ i.f.f. $s_1^\circ = s_2^\circ$ (Corollary 1.2)

$s_1^\circ = s_2^\circ$ i.f.f. $\Delta A_1 = \Delta A_2 = 0$ (Corollary 1.1)

$D_1 = D_2$ i.f.f. $\Delta A_1 = \Delta A_2 = 0$.

(b) $D_1 < D_2$ i.f.f. $s_1^\circ < s_2^\circ$ (Corollary 1.2)

$s_1^\circ < s_2^\circ$ i.f.f. $\Delta A_1 > 0$, $\Delta A_2 < 0$ (Corollary 1.1)

$D_1 < D_2$ i.f.f. $\Delta A_1 > 0$, $\Delta A_2 < 0$. Q.E.D.

Given $\hat{s}_1 = \hat{s}_2$, the corollary states that whenever two firms have identical debt ratios, a merger will not bring about a change in the agency costs of either firm. A merger would, therefore, be of no benefit.

For firms with differing debt ratios, a merger would bring about a decrease in agency costs for the firm with the larger debt ratio (firm 2) and an increase in agency costs for the other firm (firm 1). Thus, firm 2 stands to gains from a merger and firm 1 stands to lose. As was argued after Corollary 1.1, it is natural to identify firm 2 as the "acquiring" firm and firm 1 as the "acquired" firm on the basis of the sign of the merger-induced increase in agency costs. Thus, one empirical implication of the model is that acquiring firms would be characterized by higher debt ratios than the firms they acquire. As discussed in Chapter IX, quite a few research studies using a variety of statistical techniques provide evidence consistent with this prediction.

It is possible to characterize $s°$ by another easily understood ratio--the debt capacity utilization ratio U, where

$$(40)\quad U = \frac{B(F)}{B(\bar{F})}$$

Using the equation (11) expression for B(F) and the equation (24) expression for $B(\bar{F})$, equation (40) can be rewritten

$$U = \frac{\frac{b}{\bar{s}}(\bar{s} - s°)(s° - \hat{s})}{\frac{b}{4\bar{s}}(\bar{s} - \hat{s})^2}$$

$$(41)\quad U = \frac{4(\bar{s} - s°)(s° - \hat{s})}{(\bar{s} - \hat{s})^2}$$

In order to determine whether U is increasing or decreasing in $s°$,

$$\partial U/\partial s° = \frac{4}{(\bar{s} - \hat{s})^2}[(\bar{s} - s°) - (s° - \hat{s})]$$

$$(42)\quad \partial U/\partial s° = \frac{4}{(\bar{s} - \hat{s})^2}(\bar{s} - \hat{s} - 2s°)$$

From equation (42), $\partial U/\partial s°$ will be positive for

$$s° < \frac{\bar{s} + \hat{s}}{2}$$

Since it is easy to show[15] that $s°(\bar{F}) = (\bar{s} + \hat{s})/2$, and since $s°(\bar{F})$ is the largest value of $s°$ that a firm would reasonably set, in the relevant range of $s°$ values ($s° < s°(\bar{F})$),

$$(43)\quad \partial U/\partial s° > 0 \quad (\text{for } s° < s°(\bar{F}))$$

The debt utilization ratio is proportional to debt value B(F), with factor of proportionality $1/B(\bar{F})$. Therefore, U behaves much like debt value B(F) itself, initially increasing as s° increases, up to the point where $s^\circ = s^\circ(\overline{F})$, after which point increased promised payments reduce B(F) and U.

Based on the above, s° can be related to U by the following corollary.

<u>Corollary 1.4:</u>

Given $\hat{s}_1 = \hat{s}_2$,

(a) $s_1^\circ = s_2^\circ$ if and only if $U_1 = U_2$.

(b) $s_1^\circ < s_2^\circ$ if and only if $U_1 < U_2$.

<u>Proof:</u> The proof parallels that provided for D in Corollary 1.2. From equation (41), $U(\hat{s}, s^\circ) = \frac{4(\bar{s} - s^\circ)(s^\circ - \hat{s})}{(\bar{s} - \hat{s})^2}$. Fix the value of $\hat{s}$ at $\hat{s} = \hat{s}_1 = \hat{s}_2$. From equation (43), it is clear that $U(\hat{s}, .)$ is a strictly increasing function of s° (over the relevant range of s°). Therefore, $s_1^\circ = s_2^\circ$ if and only if $U(\hat{s}, s_1^\circ) = U(\hat{s}, s_2^\circ)$. Similarly, $s_1^\circ < s_2^\circ$ if and only if $U(\hat{s}, s_1^\circ) < U(\hat{s}, s_2^\circ)$. Q.E.D.

For firms with identical $\hat{s}$, the corollary establishes the firm's debt capacity utilization ratio U as a real-world referent for s°.

In Corollary 1.5, a direct link is established between the debt capacity utilization ratio and the merger-induced increases in agency costs ΔA_1 and ΔA_2.

<u>Corollary 1.5:</u>

Given $\hat{s}_1 = \hat{s}_2$,

(a) $U_1 = U_2$ if and only if $\Delta A_1 = \Delta A_2 = 0$.

(b) $U_1 < U_2$ if and only if $\Delta A_1 > 0$, $\Delta A_2 < 0$.

<u>Proof:</u> The proof parallels that provided in Corollary 1.3 for D.

Given $\hat{s}_1 = \hat{s}_2$,

(a) $U_1 = U_2$ if and only if $s_1^\circ = s_2^\circ$ (Corollary 1.4)

$s_1^\circ = s_2^\circ$ if and only if $\Delta A_1 = \Delta A_2 = 0$ (Corollary 1.1)

$U_1 = U_2$ if and only if $\Delta A_1 = \Delta A_2 = 0$.

(b) $U_1 < U_2$ if and only if $s_1^\circ < s_2^\circ$ (Corollary 1.4)

$s_1^\circ < s_2^\circ$ if and only if $\Delta A_1 > 0$, $\Delta A_2 < 0$ (Corollary 1.1)

$U_1 < U_2$ if and only if $\Delta A_1 > 0$, $\Delta A_2 < 0$. Q.E.D.

The comments made about the debt ratio D after Corollary 1.3 apply with respect to the debt capacity utilization ratio U. That is, a second empirical implication of the model is that acquiring firms should be characterized by higher debt capacity utilization ratios than the firms they acquire.

Proposition 1 and its corollaries characterize conditions under which a merger will bring about a change in agency costs and, consequently, a change in the values of the component firms. The corollaries also show that when there is a change in agency costs, one firm loses (the one with the lower D and U) and the other firm gains (the one with the higher D and U). The net effect determines whether or not the merger is synergistic. The following analysis characterizes the change in total firm value due to merger.

As discussed earlier, insiders of the merged firm maximize the current value of their equity holders' claims by undertaking both investment opportunities for states $s \geq s_m^\circ$. Using equation (6), the (levered) merged firm value V_m is then

$$(44) \quad V_m = \int_{s_m^\circ}^{\bar{s}} [V_m(s) - I_1 - I_2]\, q(s)\, ds$$

The value of the merged firm can equivalently be expressed as the sum of values of the unmerged firms less the merger-induced agency cost increases, ΔA_1 and ΔA_2:

(45) $V_m = V_1 - \Delta A_1 + V_2 - \Delta A_2$

The equivalence of the equation (44) and (45) expression for V_m is easy to show. Into equation (45), substitute the equation (6) expression for V and equation (31) and (32) expressions for ΔA_1 and ΔA_2:

$$(46)\quad V_m = \int_{s_1^\circ}^{\bar{s}} [V_1(s) - I_1]\, q(s)\, ds - \int_{s_1^\circ}^{s_m^\circ} [V_1(s) - I_1]\, q(s)\, ds +$$

$$\int_{s_2^\circ}^{\bar{s}} [V_2(s) - I_2]\, q(s)\, ds - \int_{s_2^\circ}^{s_m^\circ} [V_2(s) - I_2]\, q(s)\, ds$$

$$V_m = \int_{s_m^\circ}^{\bar{s}} [V_1(s) - I_1]\, q(s)\, ds + \int_{s_m^\circ}^{\bar{s}} [V_2(s) - I_2]\, q(s)\, ds$$

$$V_m = \int_{s_m^\circ}^{\bar{s}} [V_m(s) - I_1 - I_2]\, q(s)\, ds$$

The above equation is equivalent to equation (44).

Define ΔV as the increase in firm value resulting from the merger.

(47) $\Delta V = V_m - (V_1 + V_2)$

From equation (45), it is clear that

(48) $\Delta V = -\Delta A_1 - \Delta A_2$

Synergistic mergers are defined as those with $\Delta V > 0$. In order to characterize synergistic mergers, equation (48) can be expanded using the equation (31) and (32) definitions of ΔA_1 and ΔA_2.

$$(49)\quad \Delta V = -\frac{1}{\bar{s}} \int_{s_1^\circ}^{s_m^\circ} [a_1 + b_1 s - I_1]\, ds - \frac{1}{\bar{s}} \int_{s_2^\circ}^{s_m^\circ} [a_2 + b_2 s - I_2]\, ds$$

$$\bar{s}\Delta V = -(a_1 - I_1)(s_m^\circ - s_1^\circ) - (a_2 - I_2)(s_m^\circ - s_2^\circ)$$

$$-\tfrac{1}{2}[b_1(s_m^{\circ 2} - s_1^{\circ 2}) + b_2(s_m^{\circ 2} - s_2^{\circ 2})]$$

Substituting the equation (30) definition for s_m° into the preceding equation and simplifying yields

$$(50)\ \Delta V = \frac{b_1 b_2}{\bar{s}\ (b_1 + b_2)} (s_2^\circ - s_1^\circ)[\hat{s}_1 - \hat{s}_2 + \frac{(s_2^\circ - s_1^\circ)}{2}]$$

Note that $\frac{b_1 b_2}{\bar{s}\ (b_1 + b_2)}$ is always positive. Further, $(s_2^\circ - s_1^\circ)$ must be non-negative since (1) if $s_2^\circ = s_1^\circ$, $s_2^\circ - s_1^\circ = 0$, and (2) if $s_2^\circ \neq s_1^\circ$, the firm with the larger s° is defined as firm 2 so that $s_2^\circ - s_1^\circ > 0$. The sign of equation (50) thus critically hinges on the sign of the bracketed term. From equation (50), the following sufficient conditions can be formulated for synergistic mergers ($\Delta V > 0$) and value-neutral mergers ($\Delta V = 0$).

Proposition 2:

(a) If $s_1^\circ = s_2^\circ$, then $\Delta V = 0$.

(b) If $s_1^\circ \neq s_2^\circ$ (w.l.o.g. $s_1^\circ < s_2^\circ$) and $\hat{s}_1 \geq \hat{s}_2$, then $\Delta V > 0$.

Proof:

(a) From equation (50), $\Delta V = 0$ if $s_1^\circ = s_2^\circ$.

(b) Given $s_2^\circ > s_1^\circ$ and $\hat{s}_1 \geq \hat{s}_2$, the equation (50) bracketed term $[\hat{s}_1 - \hat{s}_2 + \frac{(s_2^\circ - s_1^\circ)}{2}] > 0$. Therefore, ΔV in equation (50) is positive. Q.E.D.

The first part of the proposition states that there will be no change in value brought about by a merger between two levered firms which utilize their technologies in identical states. This is consistent with the Corollary 1.1 result that there are no changes in agency costs for such mergers. Since such a merger would not affect firm value, there is no motive to merge.

The second part of the proposition states that the merger will be synergistic if the levered firms differ in their utilizations (w.l.o.g., $s_1^\circ < s_2^\circ$), and either (1) the utilizations is the same in the all-equity case ($\hat{s}_1 = \hat{s}_2$), or (2) the ranking of utilizations reverses from the all-equity case to the levered case (w.l.o.g. $\hat{s}_1 > \hat{s}_2$). Such a reversal occurs in the example of a synergistic merger provided in Figure 3. (In Figure 3, $s_2^\circ > s_1^\circ$, while $\hat{s}_2 < \hat{s}_1$.)

If assumption (A9) is invoked, the following corollary states that mergers can never bring about a reduction in value.

Corollary 2.1:

Given assumptions (A1) to (A9), a merger will always result in a non-negative increase in value ($\Delta V \geq 0$). Further, if $D_1 \neq D_2$ (or $U_1 \neq U_2$), a merger will always result in a positive increase in value ($\Delta V > 0$).

Proof: Assumption (A9) implies $\hat{s}_1 = \hat{s}_2 = 0$. From Proposition 2, if $s_1^\circ = s_2^\circ$, $\Delta V = 0$. From Corollaries 1.2 and 1.4, if $D_1 < D_2$ (or $U_1 < U_2$), then $s_1^\circ < s_2^\circ$ (i.e., $s_1^\circ \neq s_2^\circ$). From Proposition 2, $\hat{s}_1 = \hat{s}_2$ and $s_1^\circ \neq s_2^\circ$ implies $\Delta V > 0$. Q.E.D.

Under the assumption that the firm always has zero net-present-value projects available to it in the capital markets, the corollary has the fairly strong result that mergers result in either value increases or (at worst) no change in value. In the cases where the firms have different debt ratios or debt capacity utilization ratios, a merger will be synergistic. Even though the firm with the smaller debt ratio (debt capacity utilization ratio) suffers an increase in its agency costs, the reductions in the agency costs of the other firm is large enough to make the merger value-increasing. Thus, if potential gains to merger are "properly" apportioned, each of the firms with differing debt ratios (debt capacity utilization ratios) would benefit from merger.

A numerical example of a synergistic merger between two levered firms (for which assumption (A9) holds) follows and is illustrated in Figure 4.

$V_1(s) = 10 + \frac{1}{2}s \qquad I_1 = 10 \qquad F_1 = 2 \qquad \bar{s} = 15$

$V_2(s) = 5 + \frac{1}{4}s \qquad I_2 = 5 \qquad F_2 = 3$

Using equation (29), $s_m^\circ = 20/3$. Using equation (17), $V_1 = 209/60$ and $V_2 = 40.5/60$. Using equation (44), $V_m = 325/72$. Using equation (50) $\Delta V = 16/45$. Using equations (31) and (32), $\Delta A_1 = 64/135$ and $\Delta A_2 = -112/135$. Notice that $\Delta V = -\Delta A_1 - \Delta A_2$ (equation (49)) and $\Delta V = V_m - (V_1 + V_2)$ (equation (47)).

The strong result that all mergers characterized by $\hat{s}_1 = \hat{s}_2$ and $s_1^\circ < s_2^\circ$ are beneficial merits further examination. Substituting $\hat{s}_1 = \hat{s}_2 = 0$ into equation (50), the synergy in such mergers simplifies to

$$(51)\quad \Delta V = \frac{b_1 b_2}{2\bar{s}\,(b_1 + b_2)}\,(s_2^\circ - s_1^\circ)^2$$

which is always non-negative. The merger benefits increase as the "gap" $(s_2^\circ - s_1^\circ)$ between the component firms' incentives grows:

$$(52)\quad \frac{\partial(\Delta V)}{\partial(s_2^\circ - s_1^\circ)} = \frac{b_1 b_2}{\bar{s}\,(b_1 + b_2)}\,(s_2^\circ - s_1^\circ) > 0$$

This occurs as the result of the sort of averaging of investment incentives brought about by merger, as reflected by the fact that s_m° is a strictly convex combination of s_1° and s_2°. Thus, holding

The Merged Firm's Investment Decision

(with assumption (A9))

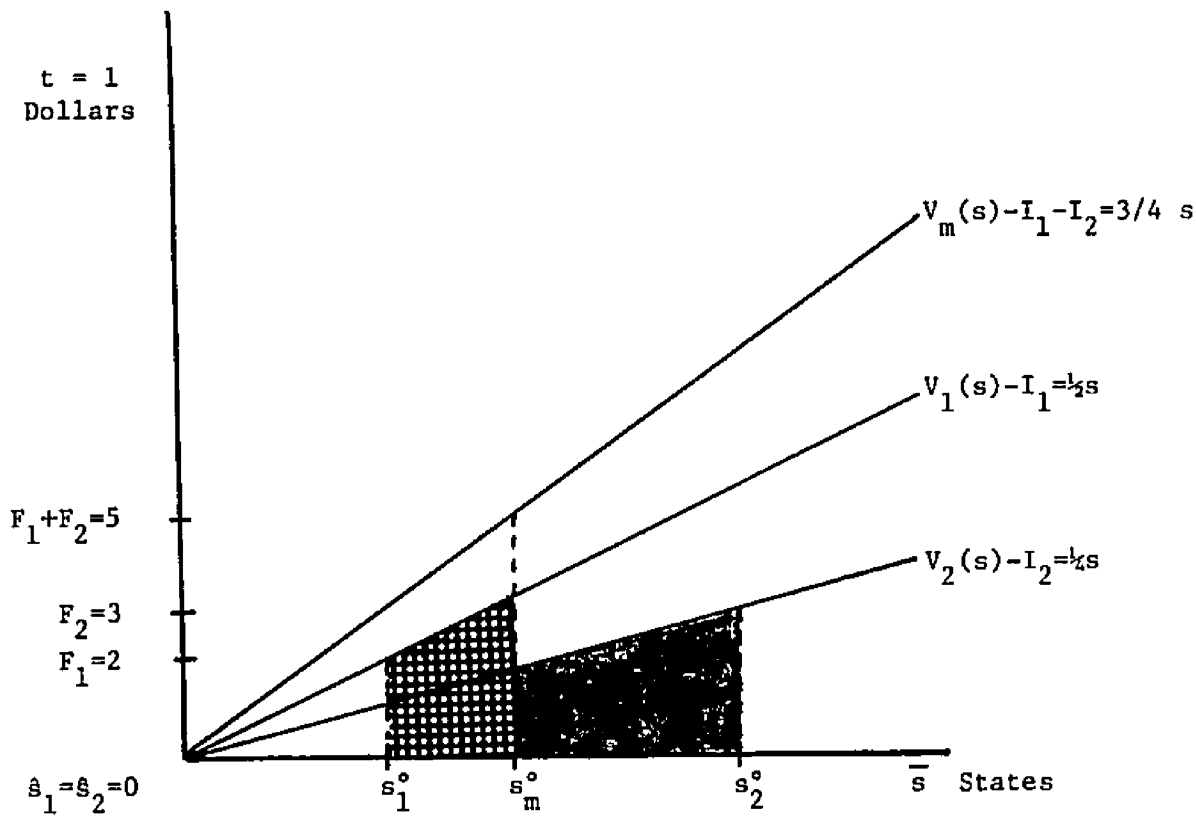

FIGURE 4

s_1° fixed, merger benefits increase in s_2° since this increases $(s_2^\circ - s_1^\circ)$. Similarly, holding s_2° fixed, merger benefits decrease in s_1° since this decreases $(s_2^\circ - s_1^\circ)$:

$$(53) \quad \frac{\partial(\Delta V)}{\partial(s_2^\circ - s_1^\circ)} = \frac{\partial(\Delta V)}{\partial(s_2^\circ)} = -\frac{\partial(\Delta V)}{\partial(s_1^\circ)} = \frac{b_1 b_2}{\bar{s}\,(b_1 + b_2)}\,(s_2^\circ - s_1^\circ) > 0$$

The quality of the firm's investment incentives, (as characterized by s°) is determined by the interaction between the firm's financial parameter F and profitability parameter b[16]. Therefore, a more fundamental analysis of the effect of mergers on investment incentives involves examination of the effects of F and b on ΔV.

Recall that under assumption (A9), $s_2^\circ = F_2/b_2$ and $s_1^\circ = F_1/b_1$, which can be substituted into equation (51) as follows:

$$(54) \quad \Delta V = \frac{b_1 b_2}{2\bar{s}\,(b_1 + b_2)}\,(F_2/b_2 - F_1/b_1)^2$$

Assuming $F_2/b_2 \neq F_1/b_1$, it must be $F_2/b_2 > F_1/b_1$ (equivalently, $s_2^\circ > s_1^\circ$). From equation (54), it can be seen that, holding other parameters fixed, ΔV increases in the promised payment level F_2 for the acquiring firm and decreases in the promised payment level F_1 for the acquired firm:

$$(55) \quad \frac{\partial(\Delta V)}{\partial F_2} = \frac{b_1}{\bar{s}\,(b_1 + b_2)}\,(F_2/b_2 - F_1/b_1) > 0$$

$$(56) \quad \frac{\partial(\Delta V)}{\partial F_1} = -\frac{b_2}{\bar{s}\,(b_1 + b_2)}\,(F_2/b_2 - F_1/b_1) < 0$$

Assume that we've properly identified firm 1 (with low s°) as the target and firm 2 (with high s°) as the bidder, and that targets are selected which maximize ΔV. Then the claim that ΔV is increasing in F_2 and decreasing in F_1 is consistent with the empirical study findings (Chapter IX) which indicate that acquiring firms tend to be heavily leveraged relative to the firms they acquire.

Retain the assumption that $F_2/b_2 \neq F_1/b_1$ (and, therefore, $F_2/b_2 > F_1/b_1$). Then from equation (54), it can be shown that, holding other parameters fixed, ΔV decreases in the acquiring-firm profitability parameter b_2 and increases in acquired-firm profitability parameter b_1:

$$(57)\quad \frac{\partial(\Delta V)}{\partial b_2} = \frac{b_1}{2\bar{s}(b_1 + b_2)} \left(\frac{F_2}{b_2} - \frac{F_1}{b_1}\right)\left[\frac{b_1}{b_1 + b_2}\left(\frac{F_2}{b_2} - \frac{F_1}{b_1}\right) - \frac{2F_2}{b_2}\right] < 0$$

$$(58)\quad \frac{\partial(\Delta V)}{\partial b_1} = \frac{b_2}{2\bar{s}(b_1 + b_2)} \left(\frac{F_2}{b_2} - \frac{F_1}{b_1}\right)\left[\frac{b_2}{b_1 + b_2}\left(\frac{F_2}{b_2} - \frac{F_1}{b_1}\right) + \frac{2F_1}{b_1}\right] > 0$$

In equations (57) and (58), the first two terms are positive; the third term in equation (57) is clearly negative, while in equation (58), it is clearly positive. Retaining the assumption that we've properly identified firm 1 as target and firm 2 as bidder, then the claim that ΔV is increasing in b_1 and decreasing in b_2 is consistent with the empirical study findings (Chapter IX) which indicate that acquired firms are more profitable than the firms that acquire them.

CHAPTER V

WEALTH TRANSFERS AMONG CLAIM HOLDERS

Chapter IV contains an examination of the effect of mergers on investment incentives, agency costs and total firm value. In this section, the focus is on how the change in total firm value is allocated among the two major groups of claimants: merged firm bondholders and merged firm stockholders.

Before exploring the pattern of claimants gains and losses brought about by merger, it is useful to detail the merged firm investment choice which drives all such effects. As was the case with the unmerged firm, merged firm equity holders cannot receive a higher return by investing and defaulting (yielding return $-I_1 - I_2 < 0$) than by refusing to invest (yielding return zero). Moreover, limited liability sharing rules are such that both sets of bondholders must be paid off before equity holders are entitled to receive any of the merged firm's cash flows. Therefore, equity could not receive more by paying off only one set of bondholders (say, firm-1 bondholders) while defaulting on the other set (firm-2 bondholders). Were they to do so, firm-2 bondholders would be entitled to receive $V_m(s) - F_1$, while equity holders receive a negative return, $- I_1 - I_2$. Again, a noninvestment strategy, which yields zero, pays a higher return than an invest-and-partial-default strategy.

Further, under fairly mild assumptions (see Appendix B), the combined cash flows are able to support the combined debt before (i.e., in a lower investment threshold state) than either project can alone. This means that either both projects will be exercised (and hence, both F_1 and F_2 repaid), or neither will. Summarizing, the investment and repayment decisions are inextricably intertwined in such a manner that insiders have only two options: (1) invest $(I_1 + I_2)$ and repay $(F_1 + F_2)$, or (2) invest zero and totally default.

Two scenarios for claimant gains and losses are possible. In the first scenario, the debt is "properly" priced (post-merger) as a zero net-present-value project. This is expected to occur if (1) the debt is issued after the merger occurs, or (2) the debt is issued pre-merger, but in perfect anticipation of the merger;

SCENARIO I

Issue Debt (any merger anticipated)	Merger Decision	State Reveals	Investment Decision	Debt Repaid	Liquidating Dividend
t=0		t=1			

Alternatively, the merger decision may precede the issuance of debt in the t=0 segment of the above time-line. In either sequence, the increase in merged firm value (ΔV) accrues to equity holders. If $\Delta V > 0$, there is an allocation of equity claims (i.e., exchange ratio) such that the equity holders of both firms will be strictly better off than they were pre-merger. In this sense, a synergistic merger is a pareto-improving step.

In the second scenario, the debt is not priced "properly" as a zero net-present-value project once the merger occurs. This is expected to be the case if the debt is issued before the merger and without bondholder anticipation of the merger and its effects:

SCENARIO II

Issue Debt (any merger not antic'd)	Merger Decision	State Reveals	Investment Decision	Debt Repaid	Liquidating Dividend
t=0		t=1			

In this scenario, it will be shown (in Corollary 3.1) that any increase in firm value (ΔV) accrues to bondholders. In addition, there is a wealth transfer from stockholders to bondholders whenever a merger changes investment incentives (again, regardless of whether the resulting ΔV is positive, negative or nets to zero). Assuming that insiders anticipate these effects, even synergistic mergers will be passed over unless strategies[17] can be devised to prevent both the wealth transfer and bondholder capture of all the synergy gains.

We initially explore the effect of improperly anticipated mergers on bond and equity values (scenario II) since the analysis will be relevant to a study of wealth effects when bonds are properly priced (scenario I). Define B_m as the aggregate merged firm bond value of promised payment $F_1 + F_2$ and ΔB as the difference between B_m and the sum of component firm bond values, B_1 and B_2:

(59) $\Delta B = B_m - B_1 - B_2$

Similarly, define E_m as the aggregate merged firm equity value and ΔE as the difference between E_m and the sum of component firm equity values, E_1 and E_2:

(60) $\Delta E = E_m - E_1 - E_2$

Recall from equation (47) that the synergy gain ΔV can be expressed as the difference between merged firm value V_m and the sum of component firm values, V_1 and V_2:

$$\Delta V = V_m - (V_1 + V_2)$$

Similar to the equation (6) identity $V = B + E$, it must also be the case that

$$(61)\quad V_m = B_m + E_m$$

Synergy gains ΔV can then be expressed as the sum of the increase in bond value ΔB and equity value ΔE. In order to do so, substitute claim values for V_m, V_1 and V_2 in equation (47) above to yield

$$\Delta V = B_m + E_m - (B_1 + E_1 + B_2 + E_2)$$

$$(62)\quad \Delta V = \Delta B + \Delta E$$

As is done for unmerged firm claim values B and E in equations (11) and (12), the merged firm claim values B_m and E_m can be solved for by integrating $t = 1$ claimant payoffs over investment states (equivalently, nondefault states) for the merged firm.

$$(63)\quad B_m = \int_{s_m^\circ}^{\bar{s}} [F_1 + F_2]\ q(s)\ ds = \frac{1}{\bar{s}}[F_1 + F_2][\bar{s} - s_m^\circ]$$

$$(64)\quad E_m = \int_{s_m^\circ}^{\bar{s}} [V_m-(I_1+I_2+F_1+F_2)]\ q(s)\ ds = \frac{b_1+b_2}{2\bar{s}}[\bar{s} - s_m^\circ]^2$$

As was the case for the unmerged firm claim values, merged firm claim values can be easily depicted. In Figure 5, merged firm bond value B_m corresponds to the shaded rectangle and merged firm equity value corresponds to the cross-hatched triangle. Merged firm agency costs correspond to the sum of the areas of (emphasized) triangles abc and dec in Figure 5.

The merged firm agency costs are shown (for component investments) in the computation of ΔA_1 and ΔA_2 (equations (31) and (32), respectively).

Merged Firm Claim Values

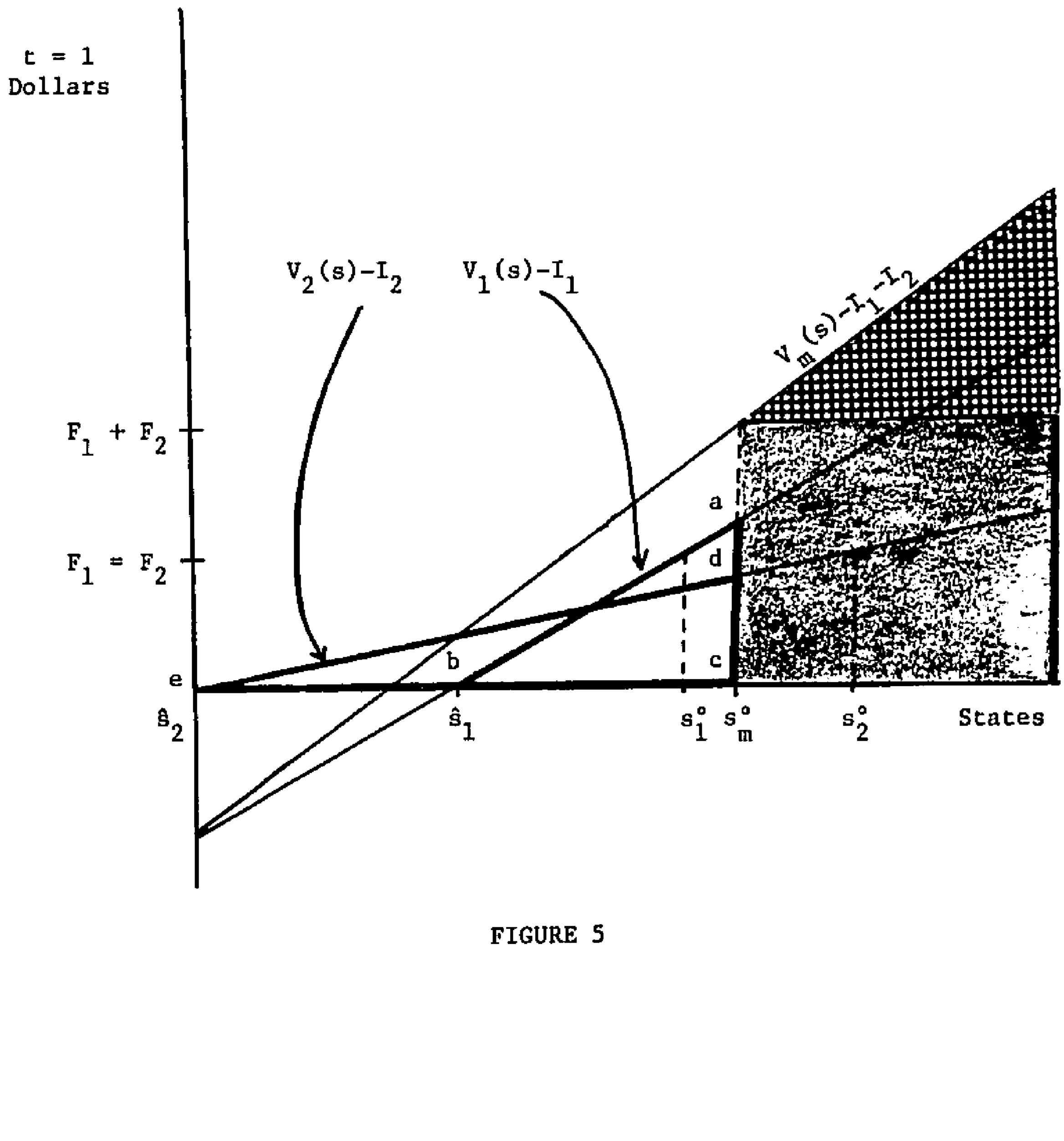

FIGURE 5

While it is unnecessary for our purposes to explicitly solve for the merged firm agency costs, their identification involves the same comparison of all-equity to levered firm value that was made earlier (in the unmerged case) in equation (7). The all-equity value of the merged firm is equal to the sum of all-equity values of the unmerged firms[18].

Comparing merged firm claim values in Figure 5 to their unmerged counterparts, it becomes apparent that the former does not involve the simple summing of unmerged claim values. While merged- and unmerged-firm claim values are based on the same limited liability sharing rules and t = 1 cash flow expressions, incentive effects are such that the merger alters the set of states over which bondholders and stockholders receive non-zero payment.

The increase in equity value ΔE can be explicitly solved for by substituting the equation (12) expression for E and the equation (64) expression for E_m into the definition of ΔE, as follows:

$$\Delta E = E_m - E_1 - E_2$$

$$\Delta E = \frac{b_1+b_2}{2\bar{s}}[\bar{s} - s_m^\circ]^2 - \frac{b_1}{2\bar{s}}[\bar{s} - s_1^\circ]^2 - \frac{b_2}{2\bar{s}}[\bar{s} - s_2^\circ]^2$$

Substituting the equation (30) definition for s_m° into the above and simplifying yields

$$\text{(65)} \quad \Delta E = \frac{-b_1 b_2}{2\bar{s}\ (b_1 + b_2)} (s_2^\circ - s_1^\circ)^2$$

From the above expression, it is clear that ΔE is never positive. This is more formally stated in Proposition 3 below.

<u>Proposition 3:</u>

(a) $s_1^\circ = s_2^\circ$ if and only if $\Delta E = 0$.

(b) $s_1^\circ \neq s_2^\circ$ (w.l.o.g., $s_1^\circ < s_2^\circ$) if and only if $\Delta E < 0$.

Proof:

(a) From equation (65), $\Delta E = 0$ if and only if $s_1^\circ = s_2^\circ$.

(b) In equation (65), the first term $\frac{-b_1 b_2}{2\bar{s}\,(b_1 + b_2)} < 0$ since $b_1 > 0$, $b_2 > 0$ and $\bar{s} > 0$; the second term $(s_2^\circ - s_1^\circ)^2 > 0$ if and only if $s_1^\circ \neq s_2^\circ$. Therefore, $\Delta E < 0$ if and only if $s_1^\circ \neq s_2^\circ$. Q.E.D.

Under the second scenario in which bondholders do not "properly" price the bond in anticipation of merger (and barring remedial strategies), the equity holders can never gain from a merger, regardless of the sign and magnitude of ΔV. At best, the merger will not affect investment incentives ($s_1^\circ = s_2^\circ$) and $\Delta E = 0$. Of course, as argued after Proposition 2, there is no motive to merge in this case (whether or not bonds are properly priced) since such a merger leaves firm value unchanged.

In the more likely event that $s_1^\circ \neq s_2^\circ$, equity holders' claims lose value regardless of whether ΔV is positive, negative or nets to zero. Barring strategies which would eliminate this wealth decrement, even synergistic mergers would be passed over.

It should be emphasized that any appreciation in firm value, including merger gains, ordinarily accrues to equity holders by virtue of their residual or ownership interest in the firm's assets. Here, not only do equity holders forfeit ΔV to bondholders--they also experience a wealth decrement ($\Delta E < 0$). Since merged-firm value is conserved, whatever equity holders lose accrues to bondholders. This is made clear by the rearrangement of equation (62) to

(66) $\Delta B = \Delta V - \Delta E$

Equation (66) provides the basis for establishing sufficient conditions for bondholder gains ($\Delta B > 0$), as shown in Corollary 3.1 below.

Corollary 3.1:

(a) $\Delta B = \Delta V + |\Delta E|$

(b) If $\Delta V \geq 0$, then $\Delta B \geq 0$.

(c) If $\Delta V > 0$, then $\Delta B > 0$.

Proof: From Proposition 3, $\Delta E \leq 0$; therefore, $-\Delta E = |\Delta E|$. Hence, equation (66) can be rewritten as $\Delta B = \Delta V + |\Delta E|$.

(b) and (c) Follow immediately from part (a) since by definition of an absolute number, $|\Delta E| \geq 0$. Q.E.D.

Part (a) states that the increase in the value of the bonds is composed of an increase in the value of the firm ($\Delta V > 0$, $\Delta V < 0$ or $\Delta V = 0$) and a non-negative wealth transfer from the equity holders equal to $|\Delta E|$. From Proposition 3, the wealth transfer will be positive whenever a merger changes investment incentives (that is, whenever $s_1^\circ \neq s_2^\circ$).

Parts (b) and (c) state that both nonsynergistic ($\Delta V = 0$) and synergistic ($\Delta V > 0$) mergers result in non-negative bondholder gains; moreover, synergistic mergers imply positive bondholder gains.

Strictly speaking, the "wealth transfer" from equity holders to debt holders consists of $\Delta V + |\Delta E|$ since, as argued after Proposition 3, all appreciation in firm value rightfully belongs to the firm's owners. Here, only ΔE is being referred to as the wealth transfer in order to emphasize that bondholders stand to gain an incremental $|\Delta E| \geq 0$ in addition to their capture of merger synergy gain ΔV. Moreover, as will be shown in Proposition 4, this wealth transfer may be so large that it "swamps" the negative effect of an overall value decline $\Delta V < 0$, leaving bondholders better off even in the event of nonsynergistic merger.

The increase in bond value ΔB can be explicitly solved for by substituting the equation (11) expression for B and the equation (63) expression for B_m into the definition of ΔB as follows:

$$\Delta B = B_m - B_1 - B_2$$

$$\Delta B = \frac{1}{\bar{s}}[F_1 + F_2][\bar{s} - s_m^\circ] - \frac{F_1}{\bar{s}}[\bar{s} - s_1^\circ] - \frac{F_2}{\bar{s}}[\bar{s} - s_2^\circ]$$

$$\Delta B = \frac{F_1}{\bar{s}}[s_1^\circ - s_m^\circ] + \frac{F_2}{\bar{s}}[s_2^\circ - s_m^\circ]$$

$$(67)\quad \Delta B = \frac{1}{\bar{s}}[F_1 s_1^\circ + F_2 s_2^\circ - (F_1 + F_2)s_m^\circ]$$

Substituting the equation (30) definition for s_m° into equation (67) and simplifying yields

$$(68)\quad \Delta B = \frac{b_1 b_2}{\bar{s}\,(b_1 + b_2)}(s_2^\circ - s_1^\circ)\left(\frac{F_2}{b_2} - \frac{F_1}{b_1}\right)$$

Based on the equation (68) expression for ΔB, the necessary and sufficient condition for $\Delta B > 0$ is established in Proposition 4.

<u>Proposition 4:</u>

Given $s_1^\circ < s_2^\circ$, $\Delta B > 0$ if and only if $\frac{F_1}{b_1} < \frac{F_2}{b_2}$.

<u>Proof:</u> Given $s_1^\circ < s_2^\circ$, the equation (68) expression for ΔB is positive if and only if $\frac{F_1}{b_1} < \frac{F_2}{b_2}$. Q.E.D.

Since the condition $\frac{F_1}{b_1} < \frac{F_2}{b_2}$ can be satisfied for all types of mergers ($\Delta V > 0$, $\Delta V < 0$, $\Delta V = 0$), it is possible (under the second scenario) for bondholders to gain from nonsynergistic and value-decreasing mergers. That is, while $\Delta V > 0$ is sufficient for $\Delta B > 0$, it is not necessary.

For an alternative interpretation of the necessary and sufficient condition for $\Delta B > 0$, recall from equation (11) that $F/b = s° - \hat{s}$. Substituting this into equation (68) and rearranging[19],

$$(69)\quad \Delta B = \frac{b_1 b_2}{\bar{s}\,(b_1 + b_2)}(s_2^\circ - s_1^\circ)[s_2^\circ - \hat{s}_2 - (s_1^\circ - \hat{s}_1)]$$

From equation (69), ΔB will be positive for $s_2^\circ - \hat{s}_2 > s_1^\circ - \hat{s}_1$--that is, bondholders gain if and only if the range of "conflict" states for the project for which incentives improve is larger than that for the project for which incentives deteriorate. Imposing assumption (A9), so that $\hat{s}_1 = \hat{s}_2 = 0$, the equation (69) expression for ΔB simplifies to

$$(70)\quad \Delta B = \frac{b_1 b_2}{\bar{s}\,(b_1 + b_2)}(s_2^\circ - s_1^\circ)^2$$

which is always positive for mergers in which investment incentives change (i.e., $s_2^\circ \neq s_1^\circ$). This is consistent with Corollary 2.1, which states all such mergers are synergistic, and Corollary 3.1, which states that synergistic mergers are sufficient for bondholder gains.

In the general case (i.e., without assumption (A9)), the relationship between ΔV and ΔB is most easily seen if ΔV is rewritten in terms of $\frac{F_2}{b_2}$ and $\frac{F_1}{b_1}$. The equation (50) expression for ΔV can be expanded to

$$\Delta V = \frac{b_1 b_2}{\bar{s}\,(b_1 + b_2)}(s_2^\circ - s_1^\circ)[\tfrac{1}{2}(\hat{s}_1 - \hat{s}_2) + \tfrac{1}{2}(s_2^\circ - \hat{s}_2) - \tfrac{1}{2}(s_1^\circ - \hat{s}_1)]$$

Substituting $F/b = s° - \hat{s}$ into the above yields

$$(71)\ \Delta V = \frac{b_1 b_2}{\bar{s}\ (b_1 + b_2)}(s_2^\circ - s_1^\circ)[\tfrac{1}{2}(\hat{s}_1 - \hat{s}_2) + \tfrac{1}{2}(\frac{F_2}{b_2} - \frac{F_1}{b_1})]$$

Assuming $s_2^\circ > s_1^\circ$, from equation (71) the necessary and sufficient condition for $\Delta V > 0$ is then $[\frac{1}{2}(\hat{s}_1 - \hat{s}_2) + \frac{1}{2}(\frac{F_2}{b_2} - \frac{F_1}{b_1})] > 0$, or $\frac{F_2}{b_2} - \frac{F_1}{b_1} > \hat{s}_2 - \hat{s}_1$.

The effects of mergers on overall firm and claimant values under the second scenario is summarized in Table I. The diagram is vertically partitioned based on the values of $\hat{s}_2 - \hat{s}_1$ and horizontally partitioned based on the values of $\frac{F_2}{b_2} - \frac{F_1}{b_1}$. The entire table is constructed assuming $s_2^\circ > s_1^\circ$, which, consistent with Proposition 3, implies equity claims lose value ($\Delta E < 0$).

If $\hat{s}_2 - \hat{s}_1 \leq 0$ (right-hand side), the merger is synergistic (Proposition 2), which in turn implies bondholders gain (Corollary 3.1). Note that assumption (A9) is a special case of $\hat{s}_2 - \hat{s}_1 \leq 0$ since (A9) implies $\hat{s}_2 = \hat{s}_1 = 0$. That is, given $s_2^\circ > s_1^\circ$, assumption (A9) implies $\Delta V > 0$ and $\Delta B > 0$, as argued after equation (70) above.

If $\hat{s}_2 - \hat{s}_1 > 0$ (left-hand side), from equation (71), the merger will be synergistic as long as the difference $\frac{F_2}{b_2} - \frac{F_1}{b_1}$ exceeds $\hat{s}_2 - \hat{s}_1$. The satisfaction of this requirement is denoted $\frac{F_2}{b_2} - \frac{F_1}{b_1} >> 0$ in Table I. Notice, however, that we can have bondholders gaining even in value-decreasing mergers ($\Delta B > 0$ and $\Delta V < 0$) if F_2/b_2 is larger than F_1/b_1, but is not sufficiently larger to satisfy $\frac{F_2}{b_2} - \frac{F_1}{b_1} >> 0$. Finally, both bond and firm value decline ($\Delta B < 0$ and $\Delta V < 0$) if $\frac{F_2}{b_2} - \frac{F_1}{b_1} < 0$.

TABLE I

Conditions for Increases and Decreases

in Claim Values

	$\hat{s}_2 - \hat{s}_1 > 0$	$\hat{s}_2 - \hat{s}_1 \leq 0$
$\frac{F_2}{b_2} - \frac{F_1}{b_1} >> 0$	$\Delta V > 0$ $\Delta B > 0$ $\Delta E < 0$	$\Delta V > 0$ $\Delta B > 0$ $\Delta E < 0$
$\frac{F_2}{b_2} - \frac{F_1}{b_1} > 0$	$\Delta V < 0$ $\Delta B > 0$ $\Delta E < 0$	
$\frac{F_2}{b_2} - \frac{F_1}{b_1} < 0$	$\Delta V < 0$ $\Delta B < 0$ $\Delta E < 0$	

The analysis thus far has dealt with aggregate wealth effects of mergers on merged firm bondholders and stockholders. Below, we consider how the merger separately affects each firm's stockholders and bondholders.

The wealth effects of merger on each firm's bondholder group is simply equal to the increase in value of promised payment F_i ($i = 1, 2$). Accordingly, define B_1^m as the value of promised payment F_1 in the merged firm. Define ΔB_1 as the merger-induced increase in the value of F_1 (i.e., as the difference between B_1^m and (unmerged) firm-1 bond value B_1):

$$(72)\quad \Delta B_1 = B_1^m - B_1$$

Similarly, define B_2^m as the value of promised payment F_2 in the merged firm and ΔB_2 as the increase in the value of F_2 brought about by merger:

$$(73)\quad \Delta B_2 = B_2^m - B_2$$

As was done for unmerged firm bond value B in equation (11), the merged firm claim value B_i^m can be solved for by integrating the promised payment F_i over nondefault (equivalently, investment) states:

$$(74)\quad B_1^m = \int_{s_m^\circ}^{\bar{s}} F_1 \; q(s)\; ds = \frac{F_1}{\bar{s}}[\bar{s} - s_m^\circ]$$

$$(75)\quad B_2^m = \int_{s_m^\circ}^{\bar{s}} F_2 \; q(s)\; ds = \frac{F_2}{\bar{s}}[\bar{s} - s_m^\circ]$$

Merged firm bond values shown above reflect the fact that both sets of bondholders are now to be paid in states $s \geq s_m^\circ$. Accordingly, bond values B_1^m and B_2^m (each) correspond to rectangle acfh in Figure 6. (In the figure, it is assumed, for convenience, that $F_1 = F_2$.)

Merger-Induced Increases and Decreases

in Claim Values

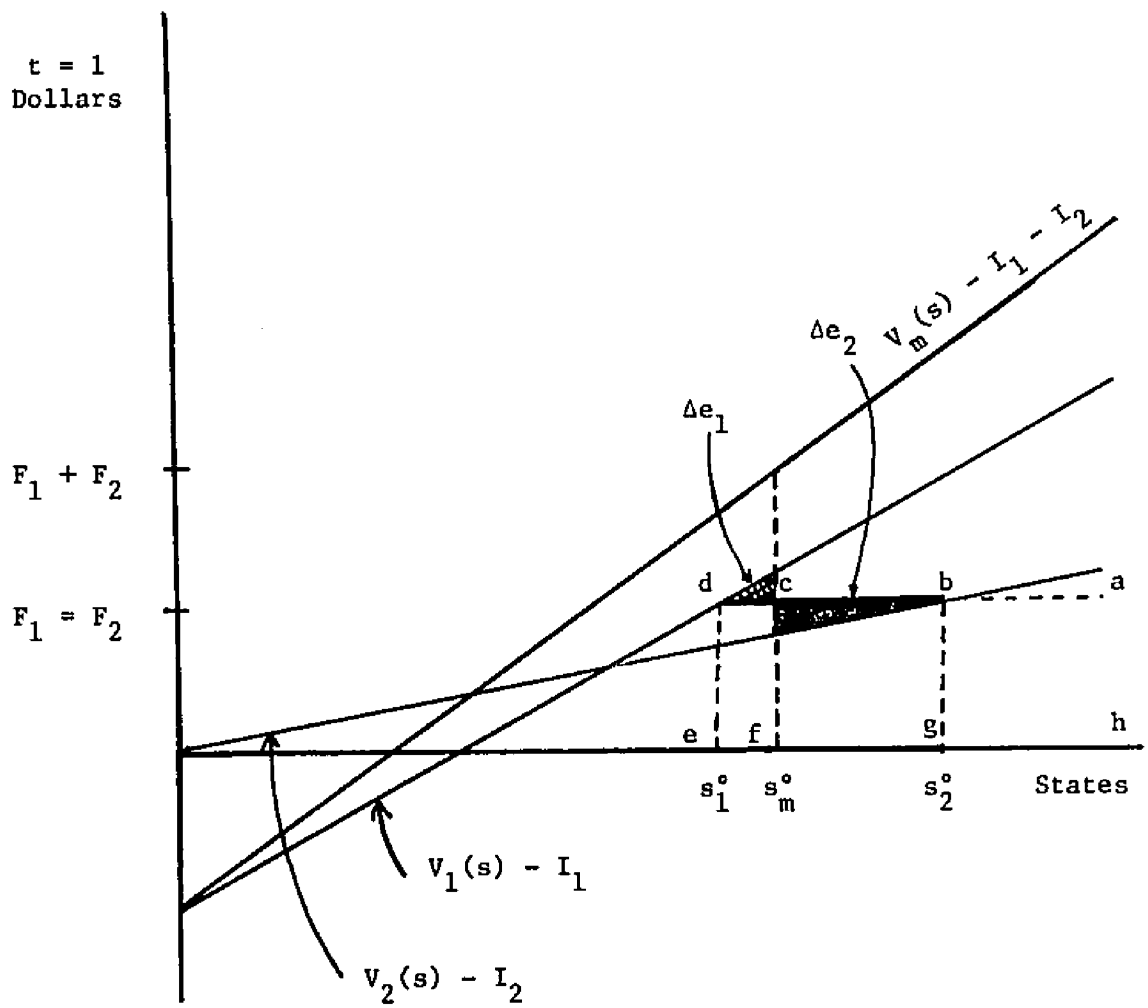

FIGURE 6

The increase in firm-1 bond value ΔB_1 can be solved for by substituting the equation (11) expression for B_1 and the equation (74) expression for B_1^m into the definition of ΔB_1, as follows:

$$\Delta B_1 = B_1^m - B_1$$

$$\Delta B_1 = \frac{F_1}{\bar{s}} [\bar{s} - s_m^\circ] - \frac{F_1}{\bar{s}}[\bar{s} - s_1^\circ]$$

$$(76)\quad \Delta B_1 = \frac{F_1}{\bar{s}} [s_1^\circ - s_m^\circ]$$

The increase in firm-2 bond value ΔB_2 can be solved for by substituting the equation (11) expression for B_2 and the equation (75) expression for B_2^m into the definition of ΔB_2 as follows:

$$\Delta B_2 = B_2^m - B_2$$

$$\Delta B_2 = \frac{F_2}{\bar{s}}[\bar{s} - s_m^\circ] - \frac{F_2}{\bar{s}}[\bar{s} - s_2^\circ]$$

$$(77)\quad \Delta B_2 = \frac{F_2}{\bar{s}}[s_2^\circ - s_m^\circ]$$

Notice that ΔB_1 and ΔB_2, specified above, sum to the aggregate increase in bond value ΔB, specified in equation (67).

In Proposition 5 below, it is shown that ΔB_1 is typically negative, while ΔB_2 is typically positive. Referring to Figure 6, it can easily be seen that ΔB_1 corresponds to rectangle cdef and ΔB_2 corresponds to rectangle bcfg. This is apparent both from the equation (76) and (77) expressions for ΔB_1 and ΔB_2, as well as from a comparison of pre- and post-merger bond values. Post-merger value B_1^m corresponds to rectangle acfh, and is thus smaller than the pre-merger value B_1, which corresponds to rectangle adeh. Post-merger value B_2^m also corresponds to rectangle acfh, and thus is larger than the pre-merger value B_2, which corresponds to rectangle abgh.

The sign of ΔB_1 and ΔB_2 can be characterized based on equations (76) and (77), respectively, as follows:

Proposition 5:

(a) If $s_1^\circ = s_2^\circ$, then $\Delta B_1 = \Delta B_2 = 0$.

(b) If $s_1^\circ < s_2^\circ$, then $\Delta B_1 < 0$, $\Delta B_2 > 0$.

Proof:

(a) Recall from Proposition I that $s_1^\circ = s_2^\circ$ implies $s_m^\circ = s_1^\circ = s_2^\circ$. Then the equation (76) and (77) expressions for ΔB_1 and ΔB_2 are identically zero ($\Delta B_1 = \Delta B_2 = 0$).

(b) Recall from Proposition I that $s_1^\circ < s_2^\circ$ implies $s_1^\circ < s_m^\circ < s_2^\circ$. Then the equation (76) expression for ΔB_1 is negative and the equation (77) expression for ΔB_2 is positive ($\Delta B_1 > 0$ and $\Delta B_2 < 0$). Q.E.D.

When the merger is not anticipated, bondholders of the second firm typically gain, while bondholders of the first firm typically lose. Referring to Table I, where it is assumed that $s_1^\circ < s_2^\circ$ throughout, it is easy to see that this pattern of (individual firm) bondholder gains and losses occurs regardless of the sign of the increase in overall bondholder value (ΔB) and firm value (ΔV).

Firm-1 bonds erode in value since repayment of F_1 is made less likely by merger. The debt is riskier because repayment is no longer made in states $s \in [s_1^\circ, s_m^\circ)$, increasing the interval of default states from $[0, s_1^\circ)$ to $[0, s_m^\circ)$. Firm-2 bonds appreciate in value since repayment of F_2 is made more likely by merger. The debt is safer because repayment is made in additional states $s \in [s_m^\circ, s_2^\circ)$, truncating the interval of default states from $[0, s_2^\circ)$ to $[0, s_m^\circ)$. As pointed out below, this reduction in default risk for firm-2 bonds is partially attributable to the coinsurance effect.

Under either pricing scenario, the share of ΔE which accrues to each shareholder group depends on the proportion of the (merged) firm that each group possesses as a result of the exchange of shares (at a prescribed rate). For example, if firm-1 stockholders hold 25% of the merged firm's shares and firm-2 stockholders hold 75% thereof, component wealth effect are $\frac{1}{4}\Delta E$ wealth decrement to firm-1 shareholders and $3/4\Delta E$ wealth decrement to firm-2 shareholders. (As shown in Chapter VI, the proper-pricing scenario involves dividing ΔV, of which ΔE is only a part, in an identical fashion.) A consideration of how the equity holders arrive at such apportionments of merger gains is deferred to Chapter VI.

While ΔE is apportioned between shareholders based on percentage of merged firm shares owned, ΔE can be meaningfully subdivided along another dimension--namely, the merger-induced increase in residual cash flows which is separately assignable to projects 1 and 2. Define Δe_i as the merger induced increase in residual cash flow from project i (i = 1, 2), where Δe_i can be computed by subtracting the pre-merger residual cash flow (E_i) from the post-merger residual cash flow for project i. That is,

$$\Delta e_1 = \int_{s_m^\circ}^{\bar{s}} [V_1(s) - I_1 - F_1]\, q(s)\, ds - E_1$$

$$\Delta e_1 = \int_{s_m^\circ}^{\bar{s}} [V_1(s) - I_1 - F_1]\, q(s)\, ds - \int_{s_1^\circ}^{\bar{s}} [V_1(s) - I_1 - F_1]\, q(s)\, ds$$

$$(78)\quad \Delta e_1 = \int_{s_m^\circ}^{s_1^\circ} [V_1(s) - I_1 - F_1]\, q(s)\, ds = -\int_{s_1^\circ}^{s_m^\circ} [V_1(s) - I_1 - F_1]\, q(s)\, ds$$

where Δe_1 is negative for $s_1^\circ \neq s_2^\circ$ since, by Proposition I, $s_1^\circ < s_m^\circ < s_2^\circ$

and $V_1(s) - I_1 - F_1 > 0$ for $s > s_1^\circ$. The negativity of Δe_1 results from the fact that project 1 is no longer exercised by the merged firm in positive residual-cash-flow states $s \in [s_1^\circ, s_m^\circ)$.

Using a parallel computation

$$\Delta e_2 = \int_{s_m^\circ}^{\bar{s}} [V_2(s) - I_2 - F_2]\ q(s)\ ds - E_2$$

$$\Delta e_2 = \int_{s_m^\circ}^{\bar{s}} [V_2(s) - I_2 - F_2]\ q(s)\ ds - \int_{s_2^\circ}^{\bar{s}} [V_2(s) - I_2 - F_2]\ q(s)\ ds$$

$$(79)\quad \Delta e_2 = \int_{s_m^\circ}^{s_2^\circ} [V_2(s) - I_2 - F_2]\ q(s)\ ds$$

where Δe_2 is negative for $s_1^\circ \neq s_2^\circ$ since, by Proposition I, $s_1^\circ < s_m^\circ < s_2^\circ$ and $V_2(s) - I_2 - F_2 < 0$ for $s < s_2^\circ$. The negativity of Δe_2 results from the fact that project 2 is now exercised by the merged firm in negative residual-cash-flow states $s \in [s_m^\circ, s_2^\circ)$.

As expected, the increases in residual cash flows for component projects add to the overall increase in equity value ΔE, defined in equation (60):

$$\Delta e_1 + \Delta e_2 = \int_{s_m^\circ}^{\bar{s}} [V_1(s)-I_1-F_1]q(s)ds + \int_{s_m^\circ}^{\bar{s}} [V_2(s)-I_2-F_2]q(s)ds - E_1 - E_2$$

$$= \int_{s_m^\circ}^{\bar{s}} [V_m(s)-(I_1+I_2+F_1+F_2)]\ q(s)\ ds - E_1 - E_2$$

$$= E_m - E_1 - E_2$$

$$= \Delta E$$

Referring to Figure 6, it is easily seen that Δe_1 corresponds to the cross-hatched triangle and Δe_2 corresponds to the shaded triangle. This is apparent from the equation (78) and (79) expressions for Δe_1 and Δe_2, in which the post-merger residual cash flow for each project is compared to its pre-merger counterpart, E. It will be argued below that Δe_2 is the wealth transfer from stockholders of the merged firm to the debt holders holding bonds with promised payment F_2. Before doing so, it is necessary to delineate the operation of the coinsurance effect in our particular model of mergers.

Recall from Chapter II that the necessary and sufficient condition for the coinsurance effect to occur is that there is at least one state in which a default by one of the merging partners coincides with a positive net equity position by the other. (In such states, the merged firm may also default, but bondholders of the otherwise defaulting firm receive more than they would have in the absence of merger.)

The above necessary and sufficient condition must be somewhat extended before application to a model in which cash flows are contingent on discretionary investment. In our model, incentives are such that a merger alters the set of investment (i.e., cash-generating) states of component firms. A coinsurance criterion which asks if one firm's positive net equity could potentially contribute to another's deficiency then becomes ambiguous.

Specifically, before merger, firm 1 has a positive net equity position $[V_1(s) - I_1 - F_1 > 0]$ coinciding with a firm-2 deficiency $[F_2 - (V_2(s) - I_2) < 0]$ in states $s \in (s_1^\circ, s_2^\circ)$. Under the usual definition, then, one might argue that in the event of merger, project 1 cash flows will coinsure project 2 debt for these states $s \in (s_1^\circ, s_2^\circ)$.

However, as explained at the start of the chapter, in the event of merger, project 1 will only be exercised in conjunction with the exercise of project 2 (and with the repayment of both F_1 and F_2). As a result of this "all or nothing" investment/repayment choice, project 1 will only be exercised by the merged firm in states for which any accompanying project-2 debt deficiency will be covered. For states $s \in [s_1^\circ, s_m^\circ)$, project 1 is not up to this task: its positive net cash flow $V_1(s) - I_1 - F_1 > 0$ is insufficient to cover the coinciding shortfall of $F_2 - (V_2(s) - I_2) < 0$ in (exercised) project 2. Therefore, neither project will be exercised for states $s < s_m^\circ$, where state s_m° is defined such that (see equation (28)):

$$V_1(s_m^\circ) - I_1 - F_1 = F_2 - (V_2(s_m^\circ) - I_2)$$

As shown above, at state $s = s_m^\circ$, the surplus from project 1 (L.H.S.) is just sufficient to cover the deficit on project 2 (R.H.S.). For states $s \in [s_m^\circ, s_2^\circ)$, the (growing) surplus on project 1 more than covers the (shrinking) deficit on project 2. At $s = s_2^\circ$, project-2 cash flows need no subsidy to carry F_2. Thus, from the standpoint of firm-2 bondholders, firm 1 has the potential (through merger) to coinsure their promised payment in states $s \in [s_m^\circ, s_2^\circ)$.

It has thus been shown that, while pre-merger analysis indicates that one project potentially contributes to the other's deficiency for states $s \in (s_1^\circ, s_2^\circ)$, a merger causes investment incentives to change in such a fashion that the condition is actually satisfied over a smaller interval $s \in [s_m^\circ, s_2^\circ)$. By prefixing the Higgins and Schall (1975) coinsurance criterion with the words "post-merger"[20] the ambiguity in its application to our model is eliminated.

Consistent with the revised coinsurance criterion, the coinsurance effect can now be valued by computing the present value of (state-contingent) amounts project 1 contributes towards the (otherwise deficient) payment of F_2:

$$\text{(80) "coinsurance amount"} = \left|\int_{s_m^\circ}^{s_2^\circ}[F_2-(V_2(s)-I_2)]\ q(s)\ ds\right| = |\Delta e_2|$$

The "coinsurance amount" coincides with Δe_2 since the deficit $F_2 - (V_2(s) - I_2) < 0$ which gives rise to Δe_2 is covered (coinsured) by contributions of project-1 cash flows. The decrease in project 1's residual cash flow (Δe_1), on the other hand, does not result from any inability to pay F_1 to which the second project may now contribute--rather, it results from passed-over investment opportunities. In a similar vein, firm-1 bondholders enjoy no coinsurance benefits. Moreover, their debt actually declines in value (Proposition 5) since the "all or nothing" investment/repayment choice leaves F_1 unpaid in (additional) states $s \in [s_1^\circ, s_m^\circ)$.

It should be noted that the increase in firm-2 bond value ($\Delta B_2 > 0$) is not entirely attributable to the coinsurance effect. This is graphically depicted in Figure 6, in which the shaded triangle (representing the coinsurance amount) is only part of rectangle bcfg, which represents the bond value increase ΔB_2. It is more formally shown by comparison of the integrals which solve for ΔB_2 and Δe_2. From the equation (73) definition of ΔB_2, we have

$$\Delta B_2 = B_2^m - B_2$$

$$= \int_{s_m^\circ}^{\bar{s}} F_2\ q(s)\ ds - \int_{s_2^\circ}^{\bar{s}} F_2\ q(s)\ ds$$

$$\Delta B_2 = \int_{s_m^\circ}^{s_2^\circ} F_2\ q(s)\ ds$$

Comparing the preceding integral expression for ΔB_2 with that for $|\Delta e_2|$ (equation 80), we find

$$\Delta B_2 > |\Delta e_2|$$

$$\int_{s_m^\circ}^{s_2^\circ} F_2\, q(s)\, ds > \left|\int_{s_m^\circ}^{s_2^\circ} [F_2 - (V_2(s) - I_2)]\, q(s)\, ds\right|$$

since $V_2(s) - I_2 > 0$ for $s > \hat{s}_2$ (and it must be $s_m^\circ > \hat{s}_2$, per Appendix B).

The coinsurance effect thus only partially accounts for $\Delta B_2 > 0$ and in no way explains the firm-1 bondholder loss ($\Delta B_1 < 0$) and residual cash flow decline ($\Delta e_1 < 0$). Instead, factors which determine the ordering $s_2^\circ > s_1^\circ$ also help explain these gains and losses. In general, firm 2 can be thought of as having poorer investment incentives than firm 1, due to any combination of the following factors: high required investment level (I), high promised payment level (F), and/or low profitability (parameters a, b). Of course, factors which determine investment incentives for firm 1 have just the opposite characterization --namely, (any combination of) low required investment, low debt burden and/or high profitability measures. As reflected in equation (30), a merger involves the averaging of all such firm parameters. It is therefore not surprising that firm-1 bondholders lose (and residual cash flows decline) in a merger which averages their low leverage/ high profitability profile with that of high leverage/low profit firm. Firm-2 bondholders gain, even beyond the coinsurance benefit, for the same reason.

Summarizing, it has been shown that under the mispricing scenario, bondholders in the aggregate benefit in synergistic mergers, although this involves bond appreciation for only acquiring-firm bondholders. Equity holders as a group do not gain, and the division of any loss between the two equity holder groups would depend on their respective share in the new firm. Since equity holders never gain under this scenario, even synergistic mergers would not be undertaken. Since mergers are not expected to occur under the mispricing scenario, no empirical implications are to be drawn here[21].

Nevertheless, Propositions 3 through 5 and Corollary 3.1 are useful for analyzing merger decisions under the first scenario (in which debt is either issued after merger or is otherwise properly priced in anticipation of merger) since it indicates under what conditions equity holders stand to gain from merger in the form of higher bond and firm values. Recall that the insiders' overall objective is to maximize the value accruing to equity holders from the available investment opportunity--this includes both the value of equity holders' claims and prices paid for external claims. One way insiders may be able to increase the value accruing to equity holders may be to merge and then issue debt rather than issue (an equal amount of) debt unmerged. Or, assuming the debt is properly priced in anticipation of merger, insiders may be able to increase the value accruing to equity holders by undertaking a merger which increases bond values (relative to the unmerged case). In either case, if the bonds are priced properly, bondholders pay for exactly what they get, and merger gains (ΔV) accrue to equity holders (in the form of ΔE and ΔB). Thus, under the first scenario, the type of analysis contained in Propositions 3 through 5 would be relevant for a "pro forma" computation of the effects of merger on firm and claim values. After making such pro forma comparisons, insiders would act in the interest of equity holders by taking merger decisions which maximize total firm value--both the value of residual claims and the price that bondholders would pay at $t = 0$. One way insiders can achieve this objective is by undertaking synergistic mergers in order to minimize agency costs.

By definition of a synergistic merger, it is possible to divide ΔV in a manner which leaves both equity holder groups better off than in the absence of merger. That is, the increase in the value of the acquiring firm's option more than compensates for the decrease in the value of the target firm's option. This allows the acquiring firm to offer a price in excess of V_1 to the target firm, while still retaining part of the synergy gains for its own equity holders. The issue of how the synergy gain is to be allocated cannot be solved based on existing financial paradigms. Instead, a game-theoretic approach to the division of synergy gains is explored in Chapter VI.

CHAPTER VI

GAME-THEORETIC APPROACHES TO SYNERGY GAIN ALLOCATION

By definition, synergistic mergers have the potential to benefit the shareholders of both merging firms. The decision of whether or not to go forth with synergistic mergers hinges on finding a sharing arrangement which is not only mutually beneficial, but is also considered "fair." This chapter contains a search for "fair" sharing arrangements using game-theoretic paradigms. Several game-theoretic solutions are identified and compared to synergy gain splits documented in the empirical literature.

This chapter is self-contained insofar as the analysis is applicable to any synergistic merger[22], regardless of the source and type (i.e., financial or operational) of synergy. A general mathematical description of the distribution of synergy gains in a 100% pure-exchange merger necessarily precedes any game-theoretic treatment. The synergy gain allocation is shown to uniquely determine (1) the appropriate adjustment of share prices at the time of the merger's announcement, and (2) merger agreement terms such as the exchange ratio and number of shares to be newly issued in the exchange. The reaction of these merger variables to changes in the overall synergy gain and synergy gain allocation is also explored.

In order to apply the analysis of this chapter to our specific agency-cost-reduction model of mergers, additional model assumptions must be made. Specifically, in addition to assumptions (A1) through (A8), assume

(A10) Bonds are properly priced at $t = 0$ such that equity holders are able to capture all synergy gains at $t = 0$.

Referred to as scenario I in the preceding chapter, this allows acquiring-firm equity holders to "get their hands on" nonzero agency cost savings (ΔA_2) at $t = 0$ by means of a merger with a suitably-chosen partner. Acquiring-firm insiders are thus in a position to propose merger terms which are mutually beneficial: some portion of ΔA_2 will be retained by the acquirer, while (at least) compensating acquired-firm shareholders for their incremental agency cost loss (ΔA_1).

(A.11) All bond proceeds are paid out as dividends at $t = 0$ (and after any merger takes place).

Recall from Corollary 3.1 that synergy gain ΔV consists of an aggregate increase in bond value ($\Delta B > 0$) and an aggregate decrease in ex-dividend equity value ($\Delta E < 0$), relative to the no-merger case. In order for ΔV to be apportioned by means of an exchange of shares, ΔV must be reflected in (cum-dividend) equity prices. This will be the case if bond proceeds are paid out as dividends, as assumed here, or otherwise accrue to equity holders in the form of nonpecuniary consumption or a stock repurchase.

The post-merger payment of dividends at $t = 0$ allows bond proceeds/dividends to be divided between the merging parties in any desired fashion.

Incorporating assumptions (A10) and (A11), the sequence of economic events is thus:

Issue Debt (any merger anticipated)	Merger Decision	Dividends Paid	State Reveals	Investment Decision	Debt Repaid	Liquidating Dividend
t=0			t=1			

Alternatively, the merger decision may be taken before issuing the debt. These sequences were collectively referred to as scenario 1 in Chapter V. In either case, bonds are properly priced at t = 0 so that synergy gain ΔV accrues to equity holders as the net effect of

(1) t=0 dividends that are ΔB higher than in the no-merger case, and

(2) ex - dividend equity value that is ΔE lower than in the no-merger case. Assume that equity holders properly anticipate effects (1) and (2) such that cum-dividend equity prices impound ΔV at t=0. Such cum-dividend equity prices are used in the analysis below to expore the effect of 100% pure-exchange mergers on share price.

The cum-dividend value of equity is $V = B + E$, specified in equation (6), where E is the ex-dividend value of equity defined in equation (5) and B is the value of bonds specified in equation (4). For the purposes of this chapter, V will alternatively be referred to as the firm value and (cum-dividend) equity value.

The cum-dividend value of the merged firm is thus V_m, where, consistent with equation (47)

$$(81)\quad V_m = V_1 + V_2 + \Delta V$$

Equation (81) states that, before dividends, equity holders of the merged firm have claims worth the sum of the (cum-dividend) equity values of constituent firms plus the merger-induced synergy gain ΔV.

In a pure-exchange merger, the acquiring firm (firm 2) distributes additional shares to acquired-firm shareholders in exchange for all acquired-firm shares. Define N_1 as the number of outstanding equity shares of unmerged firm i ($i = 1, 2$) and ΔN_2 as the number of additional shares the acquiring firm distributes to the acquired firm's shareholders. The merged firm thus has $(N_2 + \Delta N_2)$ shares outstanding after the merger is consummated, where (acquiring) firm 2 is the surviving entity.

Assume that, in equilibrium, equity claims are properly priced to reflect firm value V. Then it is easy to solve for equilibrium (cum-dividend) share prices of the unmerged firms as

(82) $P_1 = V_1/N_1$

(83) $P_2 = V_2/N_2$

where P_1 is the equilibrium share price of firm 1 (in the absence of merger) and P_2 is the equilibrium share price of firm 2 (in the absence of merger). Similarly, before dividends, the merged firm has equilibrium share price P_2^*, where

(84) $P_2^* = \dfrac{V_m}{(N_2 + \Delta N_2)}$

Rearrange equation (84) to

(85) $V_m = P_2^*(N_2 + \Delta N_2)$

Equating the right-hand sides of equations (85) and (81), merged firm value V_m is

(86) $V_m = V_1 + V_2 + \Delta V = P_2^*(N_2 + \Delta N_2)$

Suppose that firms 1 and 2 agree to merge on the condition that ΔV is shared in the following fashion: G_1 is the synergy value to be received by target-firm shareholders, and G_2 is the synergy value to be received by bidding-firm shareholders, where

(87) $\Delta V \geq G_1 + G_2$

It is realistic to assume that each firm's shareholders prefer more wealth to less wealth. Therefore, in the analysis which follows, it is assumed that each firm's shareholders demand a non-negative synergy allocation, such that

(88) $G_i \geq 0$ where $i = 1, 2$

The assumptions[23] that G_1 and G_2 are non-negative and sum to ΔV (at most) can be summarized as

(89) $0 \leq G_1 \leq \Delta V$

(90) $0 \leq G_2 \leq \Delta V$

Assuming that ΔV is entirely distributed such that synergy shares G_1 and G_2 sum to ΔV, the equation (86) expression for merged firm equity value (V_m) can be written

(91) $V_m = V_1 + V_2 + G_1 + G_2 = P_2^*(N_2 + \Delta N_2)$

Equation (91) can be decomposed into the amount of equity value which accrues to acquired- and acquiring-firm shareholders (at t=0) as a result of the merger. By definition, the value of claims held by acquiring-firm shareholders is $V_2 + G_2$ in the event of merger. Since these holdings consist of N_2 (out of $N_2 + \Delta N_2$) shares with share price P_2^*, it must be

(92) $P_2^* N_2 = V_2 + G_2$

Similarly, the value of claims held by acquired-firm shareholders is $V_1 + G_1$ in the event of merger. Since these holdings consist of ΔN_2 (out of $N_2 + \Delta N_2$) shares with share price P_2^*, it must be

(93) $P_2^* \Delta N_2 = V_1 + G_1$

Notice that equity values accruing to acquiring- and acquired-firm shareholders (specified in equations (92) and (93)) sum to overall merged-firm equity value (specified in equation (91)).

As previously described, a pure-exchange merger is brought about by the exchange of ΔN_2 acquiring-firm shares for (all) N_1 acquired-firm shares. The exchange ratio X is then simply

(94) $X = \Delta N_2/N_1$

Since V_1, V_2, and N_2 are assumed to be known and are independent of the merger terms agreed upon, equations (92) and (93) uniquely determine P_2^* and ΔN_2 once G_1 and G_2 are specified. Further, given N_1 is common knowledge, equation (94) uniquely determines the exchange ratio X once ΔN_2 is specified. In sum, assuming (unmerged) firm parameters V_1, V_2, N_1, N_2 are known, the determination of the merger split (G_1 and G_2) uniquely solves for the new equilibrium price (P_2^*), the number of acquiring-firm shares to be issued (ΔN_2) and the exchange ratio X.

The above analysis solves for merged firm share price P_2^* based on the shareholdings of each set of equity holders upon completion of the merger. Suppose a round of trading occurs after the market learns of the merger, but before the merger's completion (i.e., the exchange of shares). Further assume that (1) equity prices of the unmerged firms adjust instantaneously to the "news" of the merger, and (2) the "news" of the merger includes the information that synergy share G_1 is to be received by the acquired firm and synergy share G_2 is to be received by the acquiring firm. In the trading interval (described above), bidding-firm share price would increase from P_2 (equation (83)) to P_2^* (equation (92)), where P_2^* equals the equity value ($V_2 + G_2$) to be received as a result of merger, divided by N_2 shares outstanding. Target-firm share price would increase from P_1 (equation (82)) to P_1^*, where P_1^* is equity value $(V_1 + G_1)$ to be received as a result of merger, divided by N_1 shares outstanding:

$$(95)\quad P_1^* = \frac{(V_1 + G_1)}{N_1}$$

In an arbitrage-free equilibrium, the ratio of share prices P_1^*/P_2^* must be the same as the rate of share exchange, X:

$$(96)\quad X = \frac{P_1^*}{P_2^*} = \frac{\Delta N_2}{N_1}$$

That is, since N_1 target-firm shares are to be exchanged for ΔN_2 bidding-firm shares, these "blocks" of shares must have the same aggregate value $(P_1^* N_1 = P_2^* \Delta N_2)$.

Equation (92) can be rearranged to solve for G_2 as

$$(97)\quad G_2 = N_2(P_2^* - V_2/N_2)$$

Using the equation (83) definition of P_2 to substitute for V_2/N_2 yields

$$(98)\quad G_2 = N_2(P_2^* - P_2)$$

Equation (98) states that the synergy gain G_2 accrues to acquiring-firm shareholders in the form of a merger-induced "appreciation" of $P_2^* - P_2$ on N_2 shares held. Similarly, equation (95) can be rearranged to solve for G_1 as

$$(99)\quad G_1 = N_1(P_1^* - V_1/N_1)$$

Using the equation (82) definition of P_1 to substitute for V_1/N_1 yields

$$(100)\quad G_1 = N_1(P_1^* - P_1)$$

Equation (100) states that (in the trading interval before the merger) the synergy gain G_1 accrues to acquired-firm shareholders in the form of a merger-induced appreciation of $P_1^* - P_1$ on N_1 shares held. Furthermore, from the equation (96) no-arbitrage condition, it is clear that $P_1^* N_1 = P_2^* \Delta N_2$. Then equation (100) can equivalently be written as

$$G_1 = P_2^* \Delta N_2 - P_1 N_1$$

$$= \Delta N_2 (P_2^* - P_1 N_1 / \Delta N_2)$$

$$(101) \quad G_1 = \Delta N_2 (P_2^* - P_1 / X)$$

Equation (101) shows that G_1 can alternatively be viewed as the product of the number of shares received times the difference in value between a merged firm share and (the number of) target shares given in exchange.

For purposes of an illustration, suppose

V_1 = \$50,000,000	V_2 = \$100,000,000
N_1 = 2,500,000	N_2 = 3,000,000
P_1 = \$20.00	P_2 = \$33.33

ΔV = \$30,000,000

Further suppose that firms 1 and 2 agree to merge on the condition that ΔV is shared in proportion to their respective pre-merger values (i.e., G_1 = \$10,000,000 and G_2 = \$20,000,000). From equation (92), the equilibrium price of the merged firm which reflects G_2 = \$20,000,000 is P_2^* = \$40. Given P_2^* = \$40, equation (93) shows that the acquiring firm must give ΔN_2 = 1,500,000 shares to the target firm to achieve G_1 = \$10,000,000. Given ΔN_2 = 1,500,000, the appropriate exchange ratio is uniquely determined by equation (94) as X = 3/5.

Assume the market learns of the merger before the final exchange of shares such that target shares are still being traded. Then equation (95) solves for $P_1^* = \$24$, which reflects $G_1 = \$10,000,000$. Consistent with the no arbitrage condition of equation (96), $P_1^*/P_2^* = 24/40 = 3/5 = X$.

In the analysis thus far, it has been assumed that the market learns (or has unbiased expectations) of the (G_1, G_2) synergy allocation that the merger entails. This spurs market participants to bid up merging firm prices to P_1^* and P_2^*. The merger terms (ΔN_2 and X) which will achieve the desired (G_1, G_2) split of ΔV are then uniquely determined. By running the analysis in reverse, observation of the market's reaction to merger overtures allows empirical researchers[24] to infer the synergy-gain split and synergy gain which the market believes the merger creates. Specifically, as shown in equations (98) and (100), a comparison of pre-announcement prices (P_1 and P_2) with post-announcement prices (P_1^* and P_2^*) can be used to solve for G_1 and G_2, and hence ΔV. Alternatively, as shown in equations (92) to (94), an observation of P_2^* and X also allows the inference of the synergy-gain split.

Returning to the illustration, suppose pre-merger parameters $P_1 = \$20$, $P_2 = \$33.33$, $N_1 = 2,500,000$ and $N_2 = 3,000,000$ are common knowledge. Then observation of post-announcement prices $P_1^* = \$24$ and $P_2^* = \$40$ allows the computation of $G_1 = \$10,000,000$ using equation (100) and $G_2 = \$20,000,000$ using equation (98). Alternatively, suppose firm 2 is observed to make an offer to acquire 100% of firm 1 in an exchange of shares at a rate of 3 firm-2 shares for 5 firm-1 shares. Given $X = 3/5$, equation (96) indicates that the offer, if successful, entails the issuance of $\Delta N_2 = 1,500,000$ additional shares. Suppose further that P_2^* adjusts upward to \$40 by the time firm-1 board approval is assured[25]. The observation that $P_2^* = \$40$ and $\Delta N_2 = 1{,}500{,}000$ indicates that $G_1 = \$10{,}000{,}000$ using equation (93) and $G_2 = \$20{,}000{,}000$ using equation (92).

Variations in the (G_1, G_2) split of synergy gains will affect the surviving-firm equilibrium price P_2^* as well as merger exchange terms ΔN_2 and X. In order to explore how merged firm price P_2^* varies with G_2, rewrite equation (92) as

$$(102)\quad P_2^* = V_2/N_2 + G_2/N_2$$

The rate of change in P_2^* for a small increase in G_2 is thus

$$(103)\quad dP_2^*/dG_2 = 1/N_2$$

which is positive for $N_2 > 0$.

The linear relationship between P_2^* and G_2 in equation (102) is depicted in Figure 7 for the relevant values $G_2 \varepsilon [0, \Delta V]$. As shown in Figure 7, P_2^* is at its minimum value (denoted $\underline{P}_2^*$) when the acquiring firm receives none of the synergy gain:

$$(104)\quad \underline{P}_2^* = P_2^*(G_2 = 0) = V_2/N_2 = P_2$$

From there, P_2^* increases monotonically in G_2, reaching its maximum value (denoted $\overline{P}_2^*$) when the acquiring firm receives all of the synergy gain:

$$(105)\quad \overline{P}_2^* = P_2^*(G_2 = \Delta V) = (V_2 + \Delta V)/N_2 = P_2 + \Delta V/N_2$$

Since an increase in G_1 decreases G_2 by an equal amount, the reaction of P_2^* to an increase in G_1 is simply the negative of its reaction to an increase in G_2. To show this, substitute the strict form of equation (87) (i.e., $G_2 = \Delta V - G_1$) into equation (102):

$$(106)\quad P_2^* = V_2/N_2 + (\Delta V - G_1)/N_2$$

The Relationship between Merged Firm Price (P_2^*) and Acquiring Firm Synergy Share (G_2)

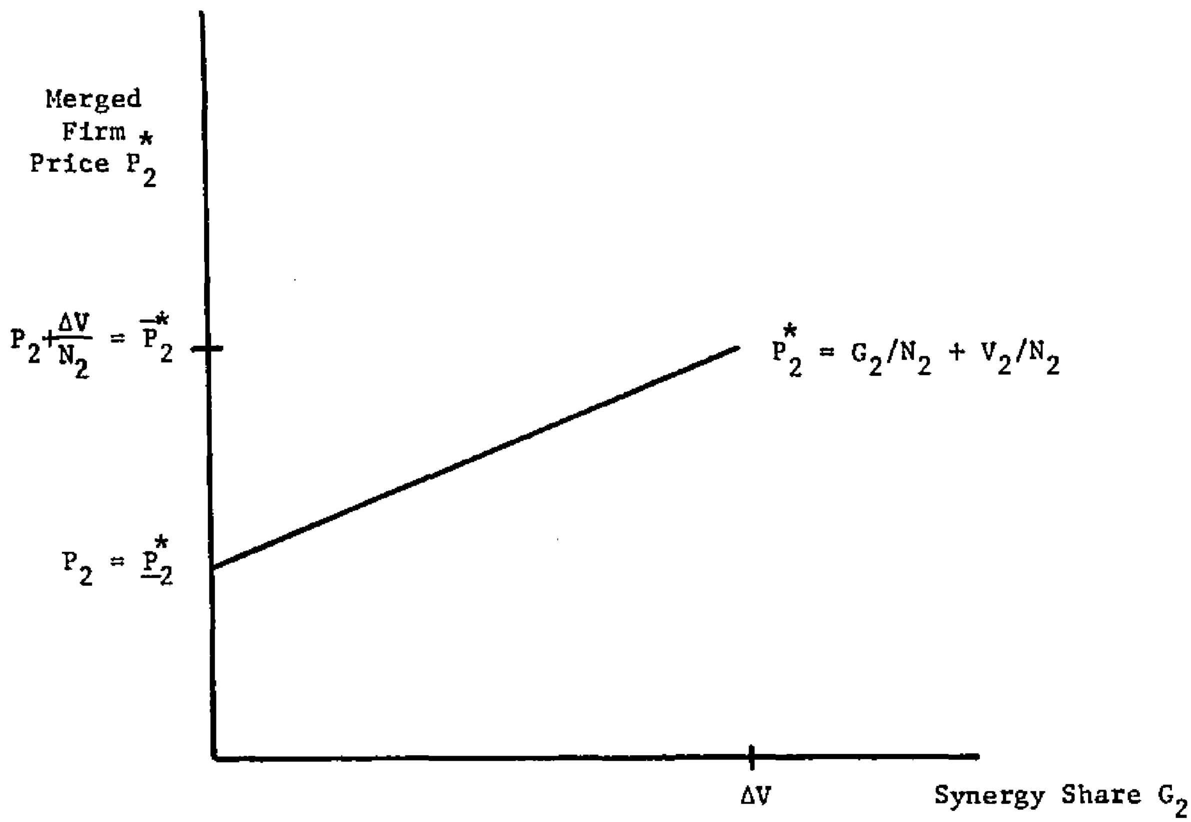

FIGURE 7

The rate of change in P_2^* for a small increase in G_1 is

(107) $dP_2^*/dG_1 = -1/N_2$

which is the negative of dP_2^*/dG_2 in equation (103).

Since P_2^* declines in G_1, the only remaining means for increasing the synergy share of the target is by increasing the number of shares acquired (ΔN_2). To show this more formally, substitute the equation (102) expression for P_2^* into equation (93) and solve for ΔN_2 as

$$(108)\quad \Delta N_2 = \frac{(V_1 + G_1)N_2}{V_2 + G_2}$$

In order to express ΔN_2 as a function of G_1 alone, substitute the strict form of equation (87) (i.e., $G_2 = \Delta V - G_1$) into the above:

$$(109)\quad \Delta N_2 = \frac{(V_1 + G_1)N_2}{(V_2 + \Delta V - G_1)}$$

To establish that ΔN_2 is increasing in G_1, solve for $d(\Delta N_2)/dG_1$ as

$$(110)\quad \frac{d(\Delta N_2)}{dG_1} = \frac{N_2(V_1 + V_2 + \Delta V)}{(V_2 + \Delta V - G_1)^2}$$

Recall from equation (47) that $V_m = V_1 + V_2 + \Delta V$. Therefore, the above derivative can be simplified to

$$(111)\quad \frac{d(\Delta N_2)}{dG_1} = \frac{N_2 V_m}{(V_2 + \Delta V - G_1)^2}$$

which is positive for the relevant range of values $N_2 > 0$ and $V_m > 0$.

The equation (109) relationship between ΔN_2 and G_1 is depicted[26] in Figure 8 over the relevant values $G_1 \in [0, \Delta V]$. As shown in Figure 8, ΔN_2 is at its minimum value (denoted $\Delta \underline{N}_2$) when the acquired firm receives none of the synergy gain:

The Relationship between the Number of Newly-Issued Shares (ΔN_2) and Acquired Firm Synergy Share (G_1)

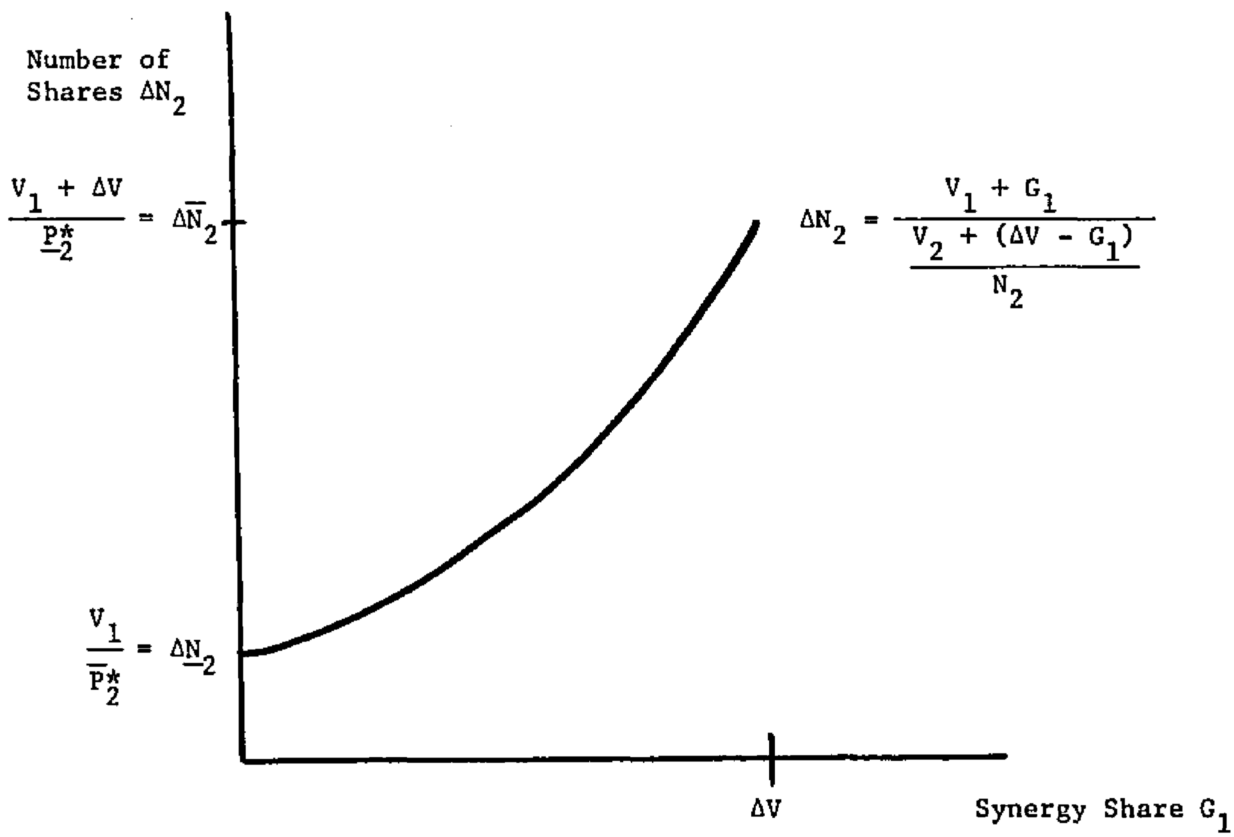

FIGURE 8

$$(112)\ \Delta\underline{N}_2 = \Delta N_2(G_1 = 0) = \frac{V_1}{\frac{V_2 + \Delta V}{N_2}} = \frac{V_1}{P_2^*}$$

From there, ΔN_2 increases monotonically in G_1, reaching its maximum value (denoted $\Delta\overline{N}_2$) when the acquired firm receives all of the synergy gain:

$$(113)\ \Delta\overline{N}_2 = \Delta N_2(G_1 = \Delta V) = \frac{V_1 + \Delta V}{\frac{V_2}{N_2}} = \frac{V_1 + \Delta V}{\underline{P}_2^*}$$

Equations (112) and (113) reflect that the determination of ΔN_2 involves the interaction of the synergy-gain split (G_1, G_2) and the merged firm price implicit in the split, P_2^*. In equation (112), the acquiring firm receives all of the synergy gain, which corresponds to P_2^* attaining its maximum value and the acquired firm receiving the minimum value (V_1) and number of shares ($\Delta\underline{N}_2$). Conversely, in equation (113), the acquired firm receives all the synergy gain, which corresponds to P_2^* reaching its minimum value and the acquired firm receiving maximum value (V_1 + ΔV) and number of shares ($\Delta\overline{N}_2$).

From the analysis above, it is clear that increases in G_1 are the net effect of increases in the number of shares acquired (ΔN_2) and decreases in P_2^*, the per-share value of shares acquired. As can be seen in equation (84), increasing the number of acquired shares spreads merged firm value V_m over more shares, thereby diluting per-share value P_2^*. Decreases in P_2^*, in turn, reduce G_2 by reducing the value of the N_2 shares held by the acquiring firm.

Since the exchange ratio $X = \Delta N_2/N_1$, and since N_1 is a positive constant, an increase in ΔN_2 (associated with an increase in G_1)

increases the exchange ratio as well. To show this more formally, divide the equation (109) expression for ΔN_2 by N_1 so that

$$(114)\ X = \frac{(V_1 + G_1)N_2}{(V_2 + \Delta V - G_1)N_1}$$

Then it is clear that the exchange ratio increases in G_1 since

$$(115)\ \frac{dX}{dG_1} = \frac{V_m N_2/N_1}{(V_2 + \Delta V - G_1)^2}$$

is positive for the relevant range of values $N_2 > 0$, $N_1 > 0$ and $V_m > 0$.

Before establishing bounds on the exchange ratio X, it is useful to use equation (95) to define minimum and maximum values of P_1^* (denoted $\underline{P}_1^*$ and $\overline{P}_1^*$, respectively) as

$$(116)\ \underline{P}_1^* = P_1^*(G_1 = 0) = V_1/N_1$$

$$(117)\ \overline{P}_1^* = P_1^*(G_1 = \Delta V) = (V_1 + \Delta V)/N_1$$

The equation (114) relationship between X and G_1 is depicted[27] in Figure 9 over the relevant range $G_1 \in [0, \Delta V]$. As shown in Figure 9, X is at its minimum value (denoted $\underline{X}$) when the acquired firm receives none of the synergy gain:

$$(118)\ \underline{X} = X(G_1 = 0) = \frac{V_1/N_1}{\frac{V_2 + \Delta V}{N_2}} = \frac{\underline{P}_1^*}{\overline{P}_2^*}$$

From there, X increases monotonically in G_1, reaching its maximum value (denoted $\overline{X}$) when the acquired firm receives all of the synergy gain.

$$(119)\ \overline{X} = X(G_1 = \Delta V) = \frac{\frac{V_1 + \Delta V}{N_1}}{\frac{V_2}{N_2}} = \frac{\overline{P}_1^*}{\underline{P}_2^*}$$

The Relationship between the Exchange Ratio (X)

and Acquired Firm Synergy Share (G_1)

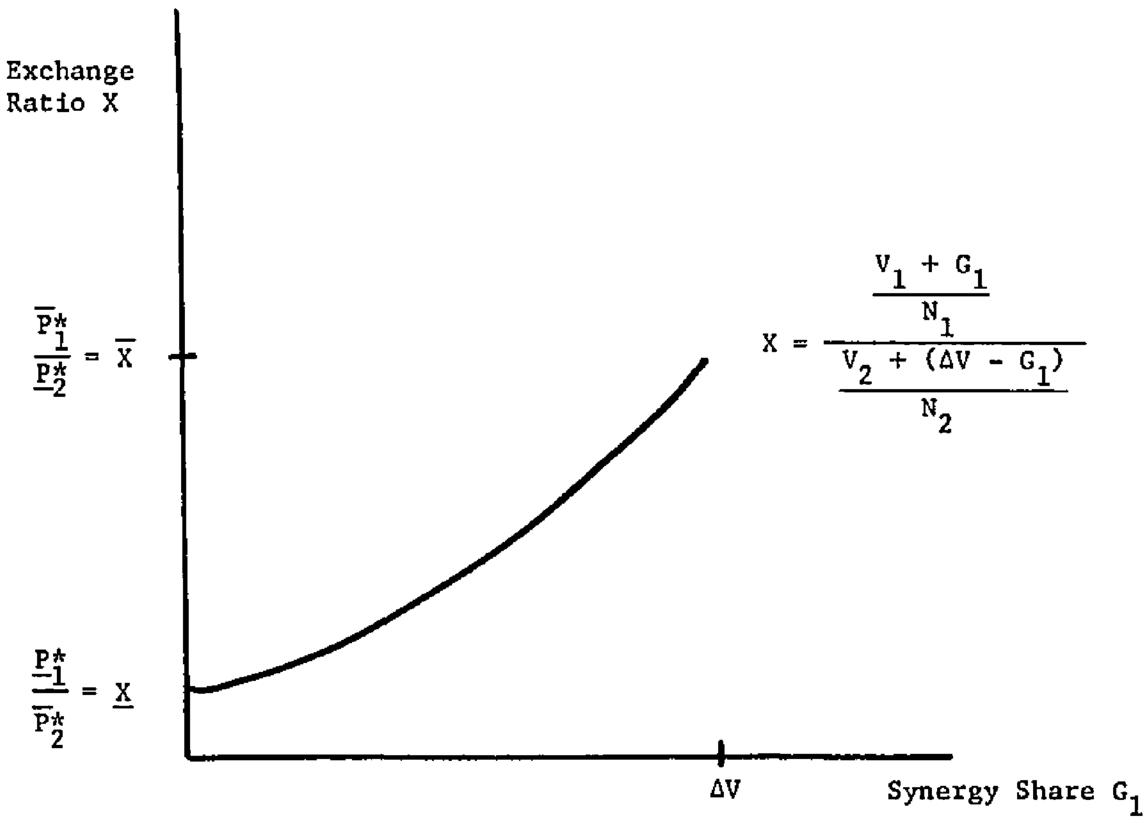

FIGURE 9

As was the case with P_2^*, it is easy to show that the reaction of ΔN_2 and X to a small increase in G_2 is simply the negative of $d(\Delta N_2)/dG_1$ and dX/dG_1, respectively. It has thus been shown that increases in synergy shares G_1 and G_2 affect equilibrium price P_2^* and merger terms ΔN_2 and X in the following fashion:

(120) $dP_2^*/dG_1 < 0 \qquad d(\Delta N_2)/dG_1 > 0 \qquad dX/dG_1 > 0$

(121) $dP_2^*/dG_2 > 0 \qquad d(\Delta N_2)/dG_2 < 0 \qquad dX/dG_2 < 0$

Since it is well-known that when a function is monotonically increasing (decreasing), the inverse function is also monotonically increasing (decreasing), equations (120) and (121) indicate that

(122) $dG_1/dP_2^* < 0 \qquad dG_1/d(\Delta N_2) > 0 \qquad dG_1/dX > 0$

(123) $dG_2/dP_2^* > 0 \qquad dG_2/d(\Delta N_2) < 0 \qquad dG_2/dX < 0$

Equations (122) and (123) characterize the changes in the allocation of gain for a small increase in P_2^*, ΔN_2 and X. Holding ΔV constant, increases in P_2^* are associated with increases in the synergy share accruing to the acquiring firm and decreases in the synergy share accruing to the target. Increases in the number of shares issued (ΔN_2) and the exchange ratio X increase the synergy share received by the target and decrease that received by the acquiring firm. The synergy share of the acquiring firm varies inversely with ΔN_2 and X since increases in ΔN_2 (and consequently, X) expand the number of merged firm shares outstanding and diminish the proportion of outstanding shares held by the acquirer $(N_2/(N_2 + \Delta N_2))$.

Having established limits for merger variables holding ΔV fixed, we now turn to a consideration of how these limits vary in ΔV. Using equation (119) to evaluate the effect of a small increase in ΔV on the maximum exchange ratio $\overline{X}$ yields

$$(124)\quad \frac{d\overline{X}}{d(\Delta V)} = \frac{N_2}{N_1 V_2}$$

which is a positive constant for the relevant ranges of values $N_1 > 0$, $N_2 > 0$, $V_2 > 0$. Using equation (118) to evaluate the effect of a small increase in ΔV on the minimum exchange ratio $\underline{X}$ yields

$$(125)\quad \frac{d\underline{X}}{d(\Delta V)} = \frac{-V_1 N_2}{N_1(\Delta V + V_2)^2}$$

which is negative for the relevant ranges of values $N_1 > 0$, $N_2 > 0$, $V_1 > 0$.

The equation (118) and (119) relationship between ΔV and the bounds on the exchange ratio is depicted in Figure 10. The upper bound $\overline{X}$ is linear in ΔV, as indicated in equation (124). The lower bound $\underline{X}$ is convex[28] in ΔV. The interval of exchange ratios between $\overline{X}$ and $\underline{X}$ can be considered a bargaining range. Any exchange ratio in this interval will not cause either set of shareholders to experience wealth dimunition as a result of the terms of the combination. If $\Delta V = 0$, the maximum and minimum exchange ratios are the same value which, consistent with the no arbitrage condition in equation (96), is equal to the ratio of pre-announcement prices P_1/P_2.

Since ΔN_2 and X are proportional (with factor of proportion N_1), any analysis of the relationship between $\Delta\overline{N}_2$, $\Delta\underline{N}_2$ and ΔV will parallel that for the exchange ratio limits and ΔV. An analysis of the relationship between ΔN_2 and ΔV is therefore omitted here.

As described earlier, in a pure exchange merger, the acquiring firm shareholders receive a $N_2/(N_2 + \Delta N_2)$ percentage ownership interest in the merged firm. Acquired firm shareholders receive the remaining $\Delta N_2/(N_2 + \Delta N_2)$ proportion of the merged firm. The values of these

The Relationship Between the Exchange Ratio Limits

and the Synergy Gain ΔV

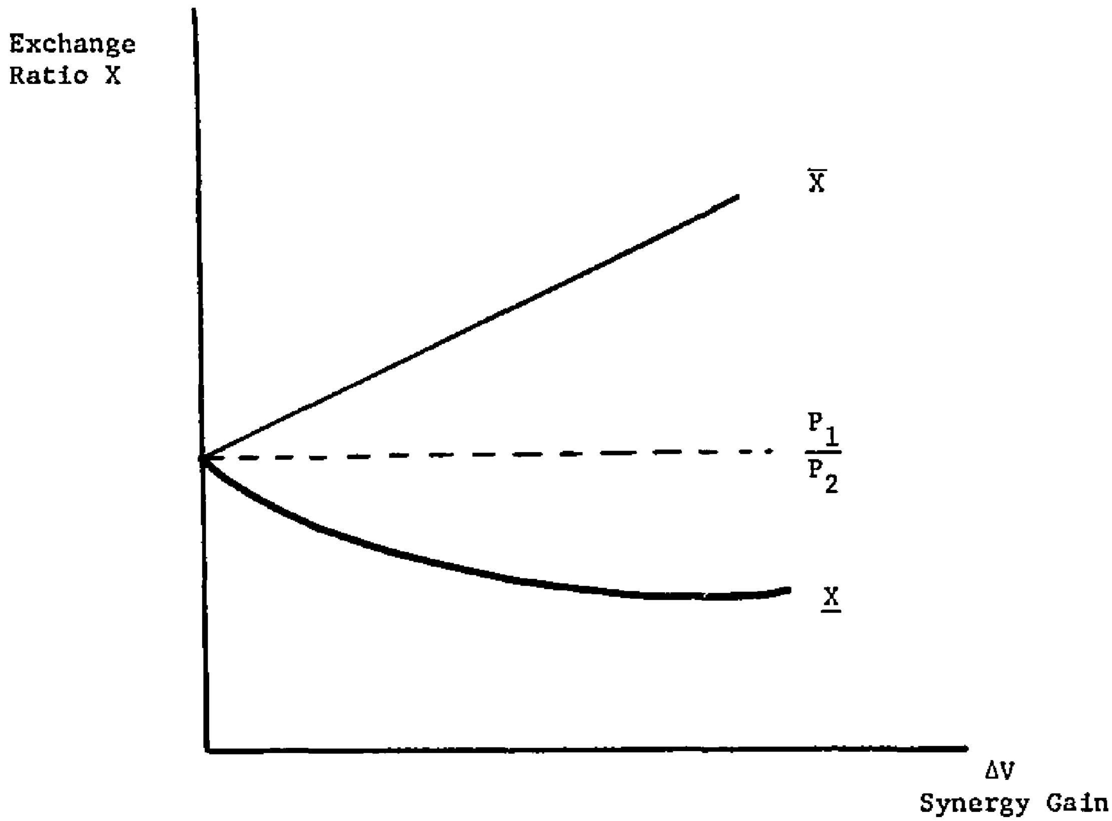

FIGURE 10

ownership interests, denoted V_2^m and V_1^m, respectively, are

$$(126)\quad V_2^m = \frac{N_2}{N_2 + \Delta N_2}(V_m)$$

$$(127)\quad V_1^m = \frac{\Delta N_2}{N_2 + \Delta N_2}(V_m)$$

where V_i^m is the cum-dividend equity value accruing to shareholders of firm i (i = 1, 2). (By definition of G_2 and G_1 as the respective gains received through merger, it must be the case that $V_2^m = V_2 + G_2$ and $V_1^m = V_1 + G_1$.)

Recall from assumption (A11) that dividends in the amount B_m are distributed after the merger has been accomplished. Then by substituting the equation (61) identity $V_m = B_m + E_m$ in equations (126) and (127) above, it is clear that the post-merger holdings V_i^m consist of dividend distributions and ex-dividend equity claim values which are proportionate to the merging party's respective ownership interest:

$$(128)\quad V_2^m = \frac{N_2}{N_2 + \Delta N_2} B_m + \frac{N_2}{N_2 + \Delta N_2} E_m$$

$$(129)\quad V_1^m = \frac{\Delta N_2}{N_2 + \Delta N_2} B_m + \frac{\Delta N_2}{N_2 + \Delta N_2} E_m$$

where B_m and E_m are specified in equation (63) and (64).

It was argued at the end of Chapter V that the mispricing scenario has implications for the correct pricing scenario (assumed here) since the former indicate how claim values are affected by the merger. The relevance of the mispricing analysis is made explicit in equations (128) and (129) since insiders for firms 1 and 2 will be making the merger decision based on how much higher dividends will be (and how much lower ex-dividend equity value will be) as a result of merger. For example, assuming all bond proceeds are paid out even in the absence of merger, firm-2 shareholders receive a dividend increase equal to

$$\frac{N_2}{N_2 + \Delta N_2} B_m - B_2$$

as a result of the merger. Similarly, firm-1 shareholders receive a dividend increase equal to

$$\frac{\Delta N_2}{N_2 + \Delta N_2} B_m - B_1$$

as a result of the merger. (Taken together, then, they receive a dividend increase of $\Delta B = B_m - B_1 - B_2$.)

The analysis thus far has focused on the relationship between the synergy allocation (G_1, G_2) and the price of surviving firm shares (P_2^*), number of newly-issued shares (ΔN_2) and the exchange ratio X. An exploration of game-theoretic approaches to a "fair" or "reasonable" division of the synergy gain follows. Specifically, the bargaining theory solution provided by Nash (1950) is used as a basis for the allocation of merger synergy gains. The bargaining solution is then compared to other game-theoretic concepts (i.e., the Shapley value and core of the game).

Nash's bargaining solution belongs to the broader cooperative game category of "arbitration schemes," where an arbitration scheme is defined as:

> . . .a function, i.e., rule, which associates to each conflict, i.e., two-person non-strictly competitive game, a unique payoff to the players. This payoff is interpreted as the arbitrated or compromised solution of the game.[29]

Thus, Nash's approach allows a unique, feasible outcome to be selected as the solution of a given bargaining problem.

One criteria for evaluation of an arbitration scheme is the reasonableness or plausibility of its axiomatic foundation (if any). It is therefore useful to set up the (general) bargaining problem and its axiomatic foundation in some detail before characterizing the solution concept. The solution concept is then introduced in a theorem. Based on the theorem, the solution to the problem of dividing the synergy gain is easily characterized.

Nash's Bargaining Model. The "pure bargaining model" is one in which two players are faced with a set of feasible outcomes, any one of which can be achieved (only) by unanimous agreement. If players are unable to agree, however, a given disagreement outcome results[30]. An incentive to reach agreement exists if there are feasible outcomes which both participants prefer to the disagreement outcome. However, with the exception of the disagreement outcome, each player has the power to veto the choices of the other. Therefore, bargaining and negotiation are necessary whenever participants differ over which outcome is most preferred.

Following Nash, the general characterization of the bargaining problem is grounded in the assumption that each player's preferences over feasible outcomes can be represented by a von Neumann-Morgenstern utility function. (It is well-known that such representations have an arbitrary origin and scale). In the specific case of the merger game, the "utilities" of the participants (i.e., two sets of insiders) will be linear in their respective firm values. That is, as emphasized later in the exposition, the proper objective for each set of insiders is to maximize firm value. As such, each set of insiders will behave *as if* they were seeking to maximize a utility function which is linear in the payoffs (here, the value of cum-dividend equity holdings).

Since each player's preferences over feasible outcomes can be mirrored by a numerical utility index, let each outcome be represented as a 2-tuple of real numbers, where the i-th component is the utility of player i for the outcome in question ($i = 1, 2$). Define S as the set of all feasible outcomes, where S is a subset of R^2 (2-dimensional Euclidean space). Assuming that players may agree to randomize between outcomes if they so choose, S will be a convex set. The convexity of S follows from the fact that players with von Neumann-Morgenstern utility functions evaluate a lottery (here, a given randomized strategy) at its expected utility. The expected utility of a given randomized strategy is a weighted average of the pure-strategy utility payoffs, using (probability) weights which are positive and sum to one.

Summarizing, the bargaining game can be specified by the set $N = \{1, 2\}$ of players and a pair (S, d), where S denotes the set of all feasible utility payoffs[31] and d is the element in S corresponding to the disagreement outcome. The set S is bounded, convex and closed (i.e., it contains its boundary). It will also be assumed that there is at least one point u in S such that $u > d$ (i.e., $u_i > d_i$ for $i = 1, 2$). This assumption focuses our attention on bargaining games in which players are motivated to seek an agreement. Let B denote the set of all bargaining games which satisfy this condition (there is some u in S such that $u > d$). Finally, assume that each of the players has complete information regarding all details of the game, including the preferences of the other player.

Nash defined the solution of the bargaining problem to be the function $f{:}B \to R^2$ such that $f(S, d) \in S$ for any $(S, d) \in B$. A solution is thus a rule which assigns to each bargaining game a feasible utility payoff of the game. The "bargaining solution" so defined will alternatively be used to refer to the outcome $f(S, d)$ of a particular game (S, d) and as the function f defined over all games in B. Let an arbitrary outcome in S be denoted $u = [u_1, u_2]$, where u_1 is the payoff to player (firm) 1 and u_2 is the payoff to player (firm) 2. The bargaining solution will be denoted $f(S, d) = u^\circ = [u_1^\circ, u_2^\circ]$.

Clearly, the merger game[33] modeled in the start of this chapter has the essential elements of the pure bargaining problem. The set $N = \{1, 2\}$ indexes the two groups of insiders, each of which represents the interests of their respective firm shareholders. The insiders use firm value (equivalently, cum-dividend equity value V_i^m, $i = 1, 2$) as an index of "utility." There is no assertion of a syndicate utility function being made here. Rather, given assumption (A7) of complete capital markets, it can be shown that the proper objective of each firm's insiders is to maximize firm value[10]. This is equivalent to an assumption that each set of insiders possesses a von Neumann-Morgenstern utility function which is linear[34] in the monetary payoffs. In our merger model, the monetary payoffs are the money value of shares obtained in a 100% pure exchange merger.

If the two groups of insiders are unable to settle on a satisfactory split of merged firm value ($V_m = V_1 + V_2 + \Delta V$), no merger takes place and the firms obtain the disagreement outcome $d = [V_1, V_2]$. Insiders have a motive to negotiate a suitable merger agreement, however, since this allows the sharing of ΔV, where (by assumption) $\Delta V > 0$.

Assuming that ΔV is infinitely divisible, the set S of feasible outcomes for the merger bargaining problem is shown in Figure 11. The shaded area in Figure 11 represents payoffs attainable through

Feasible Outcomes for the

Merger Bargaining Problem

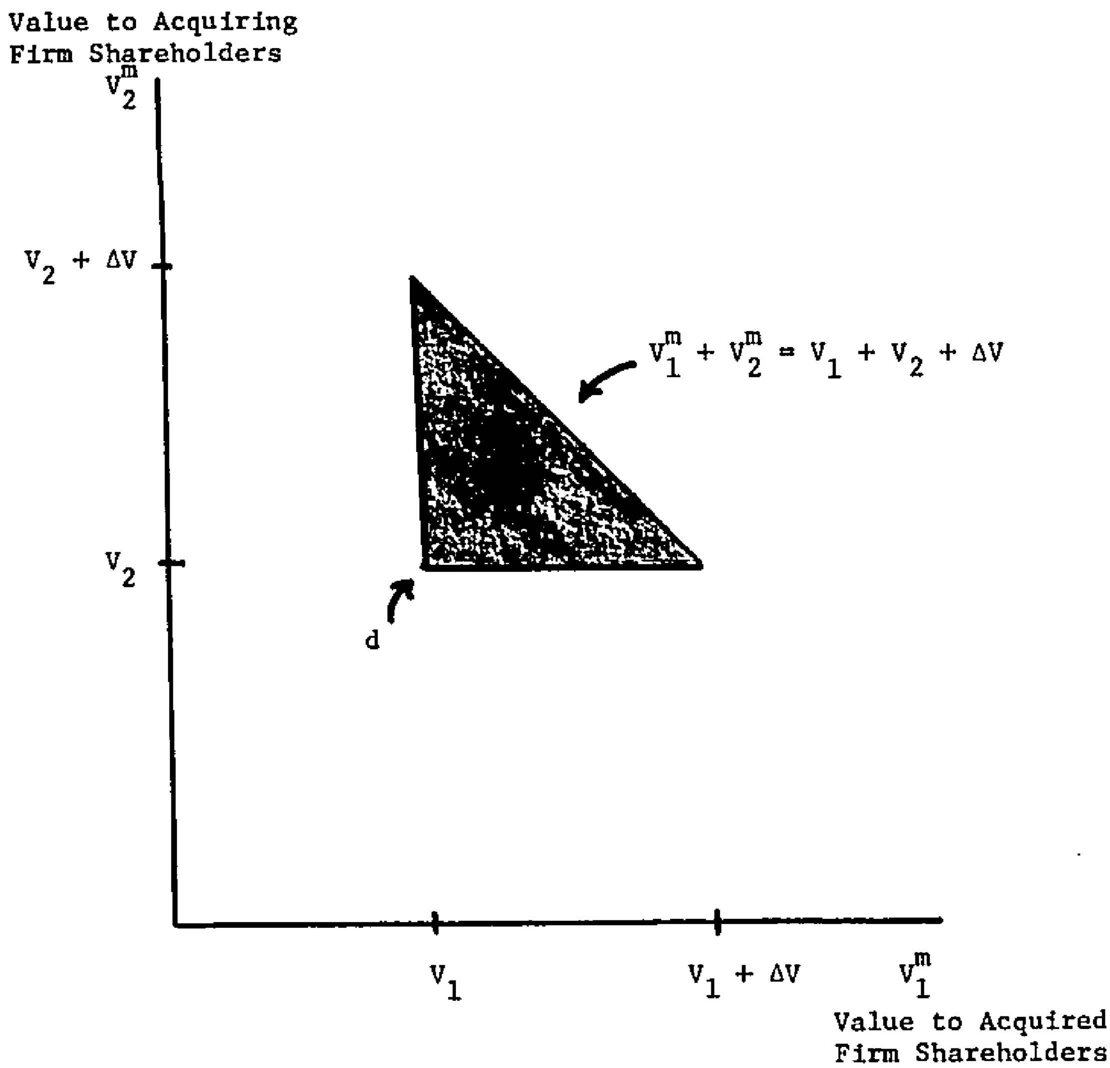

FIGURE 11

randomizations between strategies yielding payoff d and those yielding payoffs which lie along the line $V_1^m + V_2^m = V_1 + V_2 + \Delta V$. The set S can thus be characterized as:

$$(130)\quad S = \{[V_1^m, V_2^m]: V_1^m \geq V_1, V_2^m \geq V_2, V_1^m + V_2^m \leq V_1 + V_2 + \Delta V\}$$

Returning now to the (general) pure bargaining problem, Nash set forth the following axioms as a set of reasonable conditions to which any bargaining solution must conform.[32]

Axiom 1. Individual Rationality:

$$f(S, d) = \begin{bmatrix} u_1^\circ \\ u_2^\circ \end{bmatrix} \geq \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

Nash assumed that the preferences of each player can be represented by a Von Neumann-Morgenstern utility function. As such, when facing a choice between alternatives with different utility payoffs, a given player will choose the one with higher utility. Of course, the attainment of outcomes (other than the disagreement outcome) require unanimous agreement in a bargaining game. Nevertheless, a given player can always unilaterally achieve the disagreement outcome rather than agree to one which yields lower utility. Consequently, a payoff vector u with $u_i < d_i$ $(i = 1, 2)$ is not a potential solution in a game with rational players.

Axiom 2. Independence of Equivalent Utility Transformations:

For any bargaining game (S, d) and real numbers a_1, a_2, b_1, b_2 such that $a_1, a_2 > 0$, let the bargaining game (S', d') be defined by $S' = \{y \in R^2$: there exists an $x \in S$ such that $y_i = a_i x_i + b_i$ for $i = 1, 2\}$ and $d_i' = a_i d_i + b_i$ for $i = 1, 2$. Then $f_i(S', d') = a_i f_i(S, d) + b_i$ for $i = 1, 2$.

This axiom reflects the property that von Neumann-Morgenstern utility functions are only determined up to a positive linear transformation. Thus, if a given player's preferences over the possible outcomes are represented by a utility function g_i, they are equivalently represented by any utility function $h_i = a_i g_i + b_i$, where a_i and b_i are real numbers, and a_i is positive. Since the basic interests of and conflict between players is invariant with respect to the particular utility units used, the solution f should yield the same underlying outcome irrespective of the choice of g_i or h_i to represent some player's preferences.

Axiom 3. Symmetry:

If (S, d) is a symmetric bargaining game, i.e., $d_1 = d_2$ and $[u_1, u_2] \in S$ if and only if $[u_2, u_1] \in S$, then $f_1(S, d) = f_2(S, d)$.

In a symmetric bargaining game, there are no feasible payoffs with which to discriminate between the players. Since the players are thereby placed in completely symmetric roles, the solution will yield each player the same utility payoff. Thus, the solution does not distinguish between players if the abstract form of the game does not.

Nash also explains this axiom as one expressing the equality of bargaining skill of the players, which is one of the basic assumptions he makes.

Axiom 4. Independence of Irrelevant Alternatives:

Let (S, d) and (T, d) be bargaining games such that $S \subset T$ and $f(T, d) \in S$. Then $f(S, d) = f(T, d)$.

In other words, if the solution of the game with the "complete" set of alternatives is feasible in the game with a reduced set of alternatives, the solution will be the same for the two games.

Alternatively, if a feasible set is enlarged by the addition of new alternatives such that the disagreement outcome is unchanged, either the solution is unchanged or it is one of the new alternatives. The effect of Axiom 4 is to focus the attention of the bargainers on the relationship of the solution to the disagreement outcome. The axiom can be interpreted as indicating a bargaining process in which the negotiations initially involve narrowing the original set T of feasible alternatives to some smaller set S, without affecting the ultimate outcome.

Axiom 5. Pareto Optimality:

For any bargaining game (S, d), if x and y are elements of S such that $y > x$, then $f(S, d) \neq x$.

The above axiom requires that the solution f should always select an outcome from the pareto optimal frontier. The pareto optimal frontier is formed by the subset of outcomes P(S) where $P(S) \equiv \{x \in S \mid \text{there exists no } y \text{ in } S \text{ for which } y > x\}$. For example, the pareto optimal frontier in the merger game illustrated in Figure 11 consists of the line $V_1^m + V_2^m = V_1 + V_2 + \Delta V$, which forms the northeast boundary of S. The axiom can be interpreted as imposing a degree of collective rationality on the players insofar as it specifies that the solution will be an outcome such that no other feasible outcome is preferred by both of the players.

Nash's bargaining solution (described by Theorem 1 below) satisfies Axioms 1 through 5, and it is the only solution which does so.

Theorem 1:

There is a unique solution satisfying Axioms 1 to 5. It is the function F = f defined by $F(S, d) = \bar{u} = (\bar{u}_1, \bar{u}_2)$ such that $\bar{u} \geq d$ and $(\bar{u}_1 - d_1)(\bar{u}_2 - d_2) > (u_1 - d_1)(u_2 - d_2)$ for all $u = (u_1, u_2)$ in S such that $u \geq d$ and $u \neq \bar{u}$.

Thus, the Nash solution is a function which maximizes the product of the gains that the bargainers obtain by reaching an agreement instead of settling for the disagreement outcome. The formal proof of the theorem will not be presented here[35]. An outline for the mechanism of the proof is as follows. It is first argued that the solution F described in Theorem 1 satisfies Axioms 1 to 5. Then if it can be shown that Axioms 1 to 5 are satisfied by a unique solution, F must be that solution.

It is easily shown that the solution F satisfies the five axioms. For instance, the solution F satisfies Axiom 1 (individual rationality). The maximization which characterizes the solution F is

$$(131)\quad \max_{u_1,u_2} (u_1 - d_1)(u_2 - d_2) \text{ subject to } u \geq d$$

Equation (131) will choose $[u_1, u_2]$ values which are at least as large as the disagreement outcome since (1) Theorem 1 specifies $\bar{u} \geq d$, and (2) in the set B of bargaining games, there is at least one payoff $u \in S$ such that $u > d$.

The solution F also satisfies Axiom 2 (independence of equivalent utility transformations). The maximization which characterizes the solution to the transformed bargaining problem is

$$(132)\quad \max_{u_1,u_2} [a_1u_1 + b_1 - (a_1d_1 + b_1)][a_2u_2 + b_2 - (a_2d_2 + b_2)],\ a_1,\ a_2 > 0$$

Equation (132) can be simplified to

$$(133)\quad \max_{u_1,u_2} a_1a_2(u_1 - d_1)(u_2 - d_2)$$

The above maximization will choose the same $[u_1, u_2]$ values as equation (131) since a_1 and a_2 are non-negative constants. That is, $a_i F_i(S, d) + b_i = F_i(S°, d°)$ where S, S° and d, d° are described in Axiom 2. By making similar, simple arguments, it can be shown that the solution F satisfies the remaining Axioms 3 to 5.

If it can be shown that the solution f which satisfies Axioms 1 to 5 is unique, f and F must coincide. First consider the set of symmetric bargaining games. For any such game, there is only one outcome which is (both) pareto optimal and gives each player the same payoff. Axioms 3 (symmetry) and 5 (pareto optimality) require the choice of this outcome as the solution f(S, d). Thus, for symmetric games, it must be f(S, d) = F(S, d).

It remains to be shown that f(S, d) is unique for nonsymmetric games. Axiom 2 implies that if we can show a unique solution exists for a transformed game, then only one solution exists for the untransformed game. Now any game (S, d) can be transformed[36] into a normalized game (S', d') where F(S', d') = [1, 1] and d' = [0, 0]. For any normalized game (S', d'), it can be shown[37] that there is a symmetric game (A, d') for which $S' \subset A$ and [1, 1] is the pareto optimal point[38] in A. Consequently, f(A, d') = [1, 1] by Axioms 3 and 5. It must also be the case that f(S', d') = [1, 1] via Axiom 4 (independence of irrelevant alternatives) since $S' \subset A$ and $[1, 1] \in S'$. Then f(S, d) is also unique and is determined by a "reversal" of the transformations made to obtain (S', d') from (S, d). Since the solution to nonsymmetric games is unique, it must be f(S, d) = F(S, d) for such games.

To find the bargaining solution F(S, d) for the merger game represented in Figure 11, Theorem 1 suggests the following constrained optimization:

$$(134)\quad (\overline{V}_1^m, \overline{V}_2^m) = \underset{V_1^m,\, V_2^m}{\operatorname{argmax}}\ (V_1^m - V_1)(V_2^m - V_2)$$

subject to

$$(135)\quad V_1^m + V_2^m \leq V_1 + V_2 + \Delta V$$

$$(136)\quad V_1^m \geq V_1$$

$$(137)\quad V_2^m \geq V_2$$

In Appendix C, the constrained maximization described above is solved and the bargaining solution is shown to be

$$(138)\quad \overline{V}_1^m = V_1 + \tfrac{1}{2}\Delta V$$

$$(139)\quad \overline{V}_2^m = V_2 + \tfrac{1}{2}\Delta V$$

This solution corresponds to the midpoint of the line segment forming the northeast boundary of S (i.e, the pareto optimal frontier) in Figure 11.

The feasible set S of bargaining solutions depicted in Figure 11 is defined to exclude the possibility that one firm "raid" the assets of the other through merger. To be raided is defined here to mean settling on merger terms which reduce equity holder wealth (alternatively, sharing rights to assets for less than their instrinsic value). Realistic raiding scenarios can only be constructed for the bidder in tender offers[39]. Nevertheless, it is interesting to note that the addition of (symmetric or asymmetric) raiding outcomes does not alter the bargaining solution identified in equations (138) and (139).

When the possibility of raiding is proscribed, the bounds on post-merger decision value V_i^m are

$$(140)\quad V_i \leq V_i^m \leq V_i + \Delta V \qquad i = 1, 2$$

which can be seen by reference to Figure 11 or by recalling that

$V_i^m = V_i + G_i$ and $0 \leq G_i \leq \Delta V$, $i = 1, 2$ (see equations (89) and (90)). When raiding is possible, on the other hand, the raiding firm may not only capture ΔV, but also part or all of the value of the raided firm. Assuming the raided shareholders can lose their firm value at most (i.e., limited liability applies), the new bounds on post-merger decision value are

$$(141) \quad 0 \leq V_i^m \leq V_i + V_j + \Delta V \qquad i, j = 1, 2; \ i \neq j$$

Equation (141) describes a symmetric raiding scenario insofar as either firm may raid the other. The new feasible set T of outcomes (depicted in Figure 12) consists of the convex hull of $\{d\} \cup P$, where $d = [V_1, V_2]$ is the (unchanged[40]) disagreement outcome and P is the set of points satisfying

$$(142) \quad P = \{[V_1^m, V_2^m]: V_1^m + V_2^m = V_1 + V_2 + \Delta V, \ V_1^m \geq 0, \ V_2^m \geq 0\}$$

The set P thus contains the points which lie along the northeast boundary of the feasible set T (the pareto optimal frontier).

The solution to the bargaining problem with symmetric raiding opportunities is the same as that with no raiding, described in equations (138) and (139). An intuitive argument for this finding can be made by reference to Axiom 1 (individual rationality)[41]. Since the raiding scenarios yield one of the merging parties less than the disagreement outcome, raiding attempts will be repulsed.

Borrowing from the takeover literature, suppose that raiding opportunities are available only to the bidding firm. The target firm shareholders may yield part or all of their firm value to the bidder, while the reverse is no longer possible. The new bounds on post-merger decision value are

Feasible Outcomes for the Merger Bargaining Problem

with Raiding Opportunities

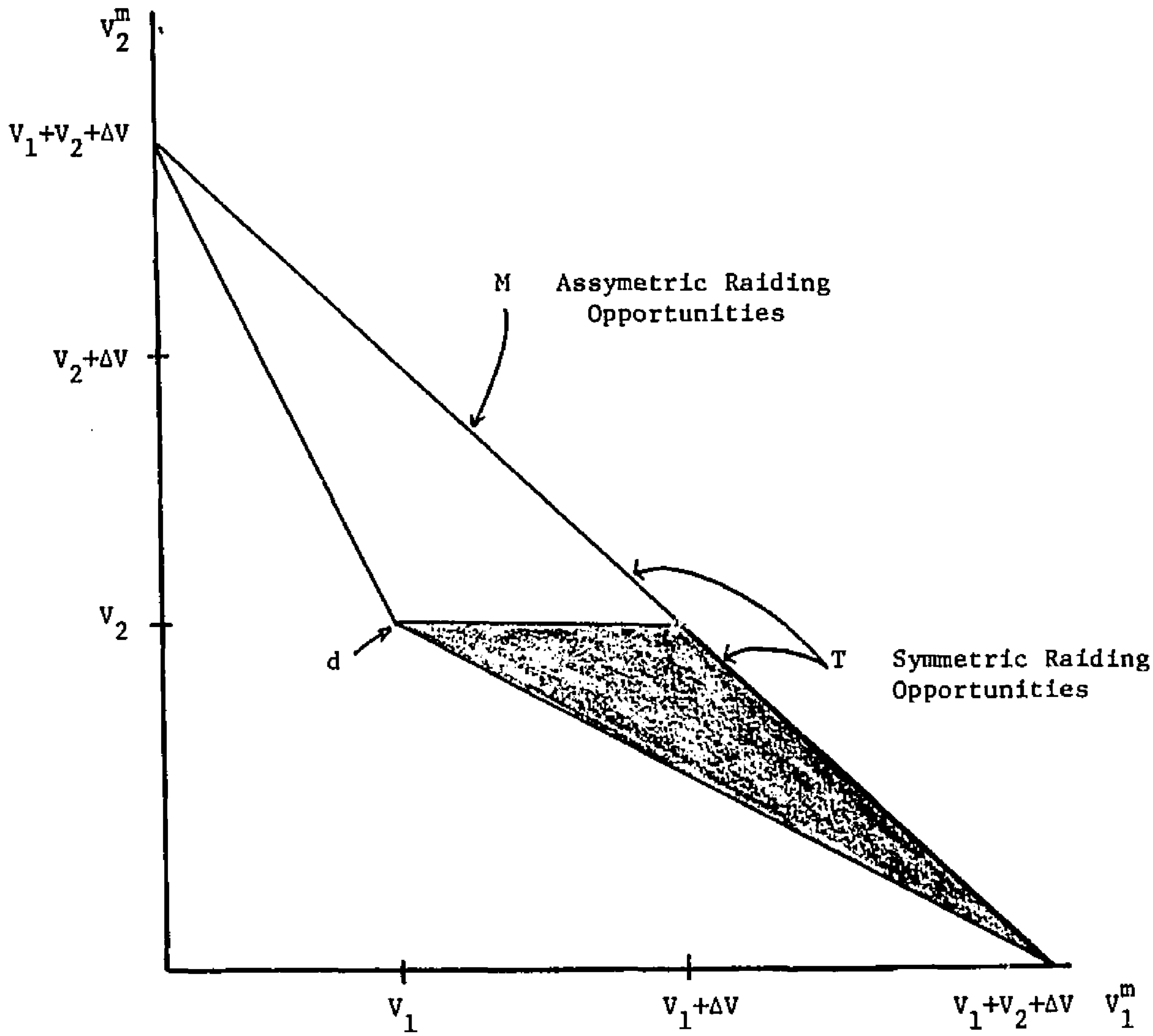

FIGURE 12

(143) $0 \leq V_1^m \leq V_1 + \Delta V$

(144) $V_2 \leq V_2^m \leq V_1 + V_2 + \Delta V$

The feasible set M of outcomes (depicted in Figure 12) consists of the convex hull of $\{d\} \cup R$, where $d = [V_1, V_2]$, as before, and R is the set of points satisfying

(145) $R = \{[V_1^m, V_2^m]: V_1^m + V_2^m = V_1 + V_2 + \Delta V,\ V_1^m \geq 0,\ V_2^m \geq V_2\}$

In Figure 12, notice that the feasible set M (asymmetric raiding) coincides with feasible set T (symmetric raiding) after elimination of the shaded wedge representing the payoffs in which firm 1 is the raider.

Consistent with Axiom 1 (individual rationality), the solution to the bargaining problem with asymmetric raiding is identical to that with symmetric raiding or no raiding[41], as described in equations (138) and (139). This finding is also made apparent if the symmetric raiding bargaining problem (with feasible set T and solution $V_i^m = V_i + \frac{1}{2}\Delta V$, $i = 1, 2$) is made the takeoff point in the analysis. The elimination of firm 1 raiding opportunities from T can then be viewed as eliminating irrelevant alternatives, which (consistent with Axiom 4) will not alter the bargaining solution. Similarly, elimination of remaining firm 2 raiding opportunities from (asymmetric raiding) feasible set M is a second application of Axiom 4, which does not alter the bargaining solution. The elimination of all raiding opportunities leaves the players with feasible set S, depicted in Figure 11, for which the equation (138) and (139) bargaining solution was originally established.

The bargaining solution can also be related to other cooperative game solution concepts. In order to do so, it is helpful to represent the merger game in characteristic form, as follows:

(146) $\nu(\{1\}) = V_1$

(147) $\nu(\{2\}) = V_2$

(148) $\nu(\{1, 2\}) = V_1 + V_2 + \Delta V$

In general, the characteristic function ν assigns to each subset S of N the maximin value of a 2-person game played between coalitions formed by the S and N-S players. The payoff obtainable by the coalition consisting of all N players is the "grand coalition" payoff, consisting here of the sum of pre-merger firm values plus the merger synergy gain.

The set of imputations for the game, denoted $E(\nu)$, consists of all payoff vectors for which (1) component payoffs sum to the grand coalition payoff, and (2) each player i receives a payoff $x_i \geq \nu(\{i\})$, $i = 1, 2$. Thus, for the merger game, $E(\nu)$ consists of the set of points which satisfy equation (136) and (137) and which lie along the line segment $V_1^m + V_2^m = V_1 + V_2 + \Delta V$, depicted in Figure 11.

The core of the game, denoted $C(\nu)$, consists of the set of all undominated imputations. As provided by Owen (1982), imputation x is said to dominate imputation y through coalition S if (1) $x_i \geq y_i$ for all $i \varepsilon$ S (each member of S prefers x to y), and (2) $\sum_{i \varepsilon S} x_i \leq \nu(S)$--i.e., the coalition is able to obtain x. It can be argued that the core is stable in the sense that there is no motivation for a subcoalition to form and isolate itself. That is, for any vector in the core, subcoalitions can find no subset of component payoffs which both pays more and is obtainable.

For the merger game, $E(\nu) = C(\nu)$ since in all 2-person games, no imputation can dominate another. To see this, suppose x and y are imputations and x dominates y through the singleton[42] consisting of player i. Then by the first dominance property, $x_i > y_i$. From the individual rationality requirement of imputations, $y_i \geq \nu(\{i\})$. Thus $x_i > \nu(\{i\})$. But this contradicts $x_i \leq \nu(\{i\})$, which is the second dominance property. Since the set of imputations and the core coincide for the merger game, $C(\nu)$ also corresponds to the set of points lying along the line segment $V_1^m + V_2^m = V_1 + V_2 + \Delta V$ depicted in Figure 11.

While the concept of imputations and the core eliminates many unacceptable outcomes in Figure 11, infinitely many synergy gain allocations remain. At the opposite extreme, in the general case, the core need not exist. These types of problems have lead to the definition of another solution concept called the Shapley value. The Shapley value can be computed for any game which can be written in characteristic form. Similar to the bargaining solution, the Shapley value belongs to the general class of arbitration schemes, for which a unique solution is identified. Also like the bargaining solution, the Shapley value may be interpreted as a "fair," mediated outcome resting on a set of "reasonable" axioms[43].

A probabilistic interpretation of the Shapley value is also possible, which is made apparent by the following formula. The Shapley value to player i for a game with characteristic function ν is

$$(149)\quad \phi_i(\nu) = \sum_{\substack{T \subset N \\ i \in T}} \frac{(t-1)!(n-t)!}{n!}[\nu(T) - \nu(T - \{i\})]$$

where t = the number of players in coalition T, n = the number of players in the grand coalition. The bracketed term may be interpreted as the incremental contribution of player i to coalition T. There are $(t-1)!(n-t)!$ ways for i to join T since, before i joins, coalition $T - \{i\}$ can be formed in $(t - 1)!$ ways and opposing coalition N-T can be formed in $(n-t)!$ ways. The marginal contribution of i to coalition T is weighted by $(t-1)!(n-t)!/n!$, which is the probability that i joins T given all permutations of coalition formation are equiprobable (i.e., have probability $1/n!$). These weighted marginal contributions are then summed over all coalitions T which i can join. Thus, the Shapley value can be viewed as paying each player his expected marginal contribution.

Equation (149) can be applied to the merger game to compute the Shapley value, denoted $\bar{\bar{V}}_i^m$ ($i = 1, 2$) for shareholders of firms 1 and 2, respectively:

$$\bar{\bar{V}}_1^m = \tfrac{1}{2}\nu(\{1\}) + \tfrac{1}{2}[\nu\{1,2\} - \nu(\{2\})]$$

$$= \tfrac{1}{2}V_1 + \tfrac{1}{2}[V_1 + V_2 + \Delta V - V_2]$$

$$(150)\quad \bar{\bar{V}}_1^m = V_1 + \tfrac{1}{2}\Delta V$$

$$\bar{\bar{V}}_2^m = \tfrac{1}{2}\nu(\{2\}) + \tfrac{1}{2}[\nu(\{1,2\}) - \nu(\{1\})]$$

$$= \tfrac{1}{2}V_2 + \tfrac{1}{2}[V_1 + V_2 + \Delta V - V_1]$$

$$(151)\quad \bar{\bar{V}}_2^m = V_2 + \tfrac{1}{2}\Delta V$$

Thus, the Shapley value of the merger game coincides with the bargaining solution specified in equations (138) and (139).

Applying equations (102), (108), and (114), it is easy to specify the stock price P_2^*, number of newly-issued shares ΔN_2, and exchange ratio X which correspond to the bargaining solution (and Shapley value) as

$$(152)\quad P_2^* = \frac{V_2 + \tfrac{1}{2}\Delta V}{N_2}$$

$$(153)\quad \Delta N_2 = \frac{V_1 + \tfrac{1}{2}\Delta V}{\dfrac{V_2 + \tfrac{1}{2}\Delta V}{N_2}}$$

$$(154)\quad X = \frac{(V_1 + \tfrac{1}{2}\Delta V)/N_1}{(V_2 + \tfrac{1}{2}\Delta V)/N_2}$$

As arbitration schemes, both the bargaining solution and the Shapley value provide a reasonable, compromise outcome (insofar as each is grounded in a well-developed axiomatic foundation). For instance, Nash interpreted the bargaining solution as a "rational expectation of gain by the two bargainers."[44] While these solution schemes are not predictive of what actually happens in bargains (for instance, they do not predict whether players arrive at an agreement or remain at the diagreement outcome), it is interesting to compare the game-theoretic merger solutions to synergy-gain allocations documented in the empirical literature.

In the merger literature, Halpern (1973) and Malatesta (1983) have computed abnormal adjusted dollar gains, while in the tender offer literature, only Bradley, Desai and Kim (1983) have used the dollar-gain approach. For a sample of 78 successful exchange mergers effected between 1950 and 1965, Halpern examines the adjusted-dollar gains for the 8-month period preceding the merger announcement. He finds that (1) the adjusted dollar gain to larger and smaller firms[45] (and total adjusted gains) are significantly positive, and (2) total adjusted gains are, on average, evenly divided between the merging firms. Thus, the (average) synergy-gain split documented by Halpern coincides with that provided by application of game-theoretic paradigms in equations (138) to (139) and (150) to (151).

For a sample of mergers occurring between 1969 to 1974, Malatesta finds that the acquiring firms experience significant losses in the period immediately before and well before the merger. Acquired firms experience significant gains in the 4 - 6 month period preceding the merger. However, these short-term gains are swamped by persistent cumulative losses which accrue 6-60 months before the merger, leaving target shareholders with an overall (5-year) insignificant dollar loss. Since the short-term losses to acquiring firms exceed the gains to acquired firms, Malatesta suggests that synergy gains in mergers may be negative. It is difficult to explain why these mergers were ever proposed or completed in the face of these results.

However, for a matched-firm subsample of 30 mergers, Malatesta's findings are closer to those in Halpern's study. There is a significant average increase of $18.6 million in the value of acquired-firm shares and an insignificant average increase of $13.8 million in the value of acquiring-firm shares. Assuming that these appreciation figures can be added to approximate an (average) overall synergy gain, this represents a 57% - 43% split, where any such split (i.e., one completely distributing ΔV) is contained in the core. Malatesta's evidence suggests that merger targets are generally able to extract a larger share of synergy gains in the merger negotiations than the bidding firms.

In their examination of 183 successful tender offers effected between 1962 and 1980, Bradley, Desai and Kim (1983) provide some evidence that tender offer targets are also able to obtain a greater share of the synergy gain than bidding firms. Unfortunately, their adjusted dollar data does not appear to have been drawn from a normal distribution; thus, they are unable to draw statistical inferences concerning dollar-gain measures. However, the abnormal return findings of Bradley, et. al., are consistent with their hypothesis that the following factors benefit takeover targets: competition of multiple bidders, federal regulations designed to protect target interests (i.e., the Williams Amendment) and the targeting of a large percentage of acquired-firm shares. These are important differences in any game-theoretic analysis of tender offers and 100% pure-exchange mergers[46]. Nevertheless, the competition and regulatory factors identified by Bradley, et. al., are suggestive of factors which would impinge on the relative bargaining position of parties to a 100% pure-exchange merger.

Summarizing, the actual bargains made by merging firms are not far from those arrived at using game-theoretic paradigms--in terms of gross (adjusted) dollars, approximately 50-60% of the synergy gain accrues to the target, and may depend on the degree of competition for target resources and the regulatory environment in which the merger occurs[47].

CHAPTER VII

THE STATE-CONTINGENT MERGER DECISION

In all the previous merger analysis (Chapters IV to VI) it has been assumed that the merger decision is taken in $t = 0$, prior to the state revelation in $t = 1$ (ex-ante merger decision). In this chapter, we'll explore how investment incentives, agency costs and claim values are affected by merger when the merger decision can be made after the state is revealed in $t = 1$ (ex-post merger decision). The sequence of economic events for the ex-post meger decision is assumed to be

Issue Debt	State Reveals	Merger Decision	Dividends Paid	Investment Decision	Debt Repaid	Liquidating Dividend
|—t=0—|	|————————	————————	———t=1——	————————	————————	————————|

A distinction should be maintained between the merger decision and the firms' "other" investment decisions based on the fact that any merger is "free" (requires no additional investment outlay), and hence does not entail the (levered-firm) investment disincentives explored in Chapter III. In addition, the merger decision is assumed to be taken in advance of the firms' "other" investment decisions, and thus has the potential to improve the incentives to undertake such investments.

Retain the proper pricing assumption (A10) made in Chapter VI in which bondholders properly anticipate the incentives of insiders to merge so that equity holders capture all synergy gains. This assumption allows merger gains to be properly reflected in (cum-dividend) share price and apportioned via an exchange of shares. Also retain an amended version of assumption (A11) made in Chapter VI whereby all bond proceeds are paid out as dividends--only now it is assumed the dividends are paid after an (ex-post) merger decision which take place at $t = 1$[48]. As was the case in Chapter VI, the post-merger payment of dividends allows merger gains (of which $\Delta B > 0$ is a major component) to be impounded in share prices and divided in any desired fashion via an exchange of shares.

In order to keep the analysis as simple as possible, it is also assumed that the dividends are paid in advance of the investment decision; otherwise, the firm possesses seizable assets which unnecessarily complicate the model in a manner described in Appendix A.

Since the insiders' objective is to maximize firm value, they will favor a merger which increases the firm value accruing to their equity holder group. An ex-post merger has the potential to create value (reduce agency costs) only if the investment decisions of the merged and unmerged firms differ. For this reason, no merger will be undertaken in the relatively poor states ($s < s_1^\circ$) and good states ($s \geq s_2^\circ$). As summarized in Table II, the stockholders of firms 1 and 2 are indifferent between the decision to merge or not to merge in these sets of states since in the former case ($s < s_1^\circ$), neither the merged firm nor the component firms will undertake either project; in the latter case ($s \geq s_2^\circ$), both the merged firm and component firms will undertake both projects.

For the interval of states $s_1^\circ \leq s < s_m^\circ$, the merged firm would be unwilling to exercise the projects since, in so doing, the combined-

TABLE II

The Ex-Post Merger Decision

State s	Unmerged-Firm Investment Decision		Pro-Forma Merged-Firm Investment Decision		Merger Incentives		Merger Decision	
	Project 1	Project 2	Project 1	Project 2	Firm 1	Firm 2	Firm 1	Firm 2
$s < s_1^\circ$	No	No	No	No	Indifferent	Indifferent	No	No
$s_1^\circ \leq s < s_m^\circ$	Yes	No	No	No	Against	Indifferent$^-$	No	No
$s_m^\circ \leq s < s_2^\circ$	Yes	No	Yes	Yes	Indifferent$^+$	For	Yes	Yes
$s_2^\circ \leq s$	Yes	Yes	Yes	Yes	Indifferent	Indifferent	No	No

Indifferent$^-$ indicates indifference or, at most, marginal opposition to a merger by the firm's insiders.

Indifferent$^+$ indicates indifference or, at most, marginal favorability to a merger by the firm's insiders.

firm debtholders could not be paid off out of net cash flows. Firm-1 shareholders, therefore, would oppose any merger for $s_1^\circ \leq s < s_m^\circ$ since they stand to lose $V_1(s) - I_1 \geq F_1$ in so doing. Firm-2 shareholders, on the other hand, are indifferent to merger for $s_1^\circ \leq s < s_m^\circ$ since project 2 will not be exercised in either the merged or unmerged firm. Moreover, if the merger-induced loss of $V_1(s) - I_1 > F_1$ were to be (partially or wholly) borne by themselves as an inducement to firm-1 shareholders, firm 2 shareholders would also oppose any merger in states $s_1^\circ \leq s < s_m^\circ$. It is therefore assumed that mergers will not occur in these states.

For the interval of states $s_m^\circ \leq s < s_2^\circ$, firm-2 shareholders would favor a merger since the merged firm will undertake positive net-present-value project 2 while the unmerged firm 2 will not. Firm-1 shareholders, ordinarily indifferent to a merger (since project 1 is exercised by the merged or unmerged firm), could be induced to merge by firm-2 shareholders if the resulting increase in firm value (defined below) were shared.

In the construction of Table II, stockholders are assumed to be "indifferent" to merger prospects in which there are no effects on either firm's investment decision, agency costs and cash flows. Stockholders are assumed to be either indifferent or marginally favorable (denoted "indifferent$^+$") to a merger in which (1) their firm' investment decision is not altered, but (2) the other firm's investment decision is improved (resulting in an agency-cost reduction, possibly to be shared). Stockholders are assumed to be either indifferent or marginally opposed (denoted "indifferent$^-$") to a merger in which (1) their firm's investment decision is not altered, but (2) the other firm's investment decision is worsened (resulting in an agency-cost increase, possibly to be shared). Finally, shareholders favor (oppose) a merger in which their firm's investment decision is improved (worsened). As shown in Table II, a merger is potentially rewarding to both sets of stockholders only in states $s_m^\circ \leq s < s_2^\circ$.

Consistent with the above analysis, a merger will only take place in states $s \in [s_m^\circ, s_2^\circ)$. In this interval of ex-post merger states, insiders maximize shareholder wealth by exercising both projects (and paying off both sets of debtholders). Merger-induced agency cost savings can now be computed by comparing the investment decisions of the merged and unmerged firms. Comparing firm-2 investment decisions to those of the ex-post merged firm, it is apparent that the merged firm exercises project 2 in additional states $s \in [s_m^\circ, s_2^\circ)$. This results in an agency cost savings on project 2 which is identical to that computed in the ex-ante merger case. The (negative) increase in agency costs is equal to ΔA_2 (equation (32)), where

$$\Delta A_2 = - \int_{s_m^\circ}^{s_2^\circ} [V_2(s) - I_2] \; q(s) \; ds$$

Investment incentives for firm 1 and the ex-post merged firm coincide for $s \geq s_m^\circ$. Since a merger does not occur for $s < s_m^\circ$, project 1 will be exercised (by firm 1 standing alone) in states $s_1^\circ \leq s < s_m^\circ$. As a result, the merger-induced agency cost increase ΔA_1 (equation (31)), which was identified for the ex-ante merger, is entirely avoided in the ex-post merger case.

Paralleling the synergy gain computation $\Delta V = - \Delta A_1 - \Delta A_2$ (equation (48)) for the ex-ante merger, the synergy gain in the ex-post merger (denoted ΔV°) is simply the negative of the agency cost increase(s) such merger induce. Therefore,

(155) $\Delta V^\circ = -\Delta A_2$

where $\Delta V^\circ > 0$ for $s_1^\circ \neq s_2^\circ$ by virtue of the fact that $\Delta A_2 < 0$ for all such mergers (see Corollary 1.1). Therefore, in the ex-post merger case, we obtain a result similar to that obtained in Corollary 2.1, in which all mergers between firms with different debt ratios (debt capacity utilization ratios) are synergistic.

Comparing equations (48) and (155), it is clear that synergy gains are greater in the ex-post merger case than in the ex-ante case ($\Delta V^\circ > \Delta V$) for $s_1^\circ \neq s_2^\circ$, since

(156) $\Delta V^\circ - \Delta V = -\Delta A_2 - (-\Delta A_1 - \Delta A_2) = \Delta A_1$

where $\Delta A_1 > 0$ for $s_1^\circ \neq s_2^\circ$ (Corollary 1.1). Thus, a merger decision made after the state reveals produces greater benefits (value) than a merger consummated between identical firms before the state reveals. The increased value is attributable to the fact that the ex-post decision enables firms to merge only in states in which it is unambiguously beneficial to do so. Mergers are avoided in "bad merger states" ($s_1^\circ \leq s < s_m^\circ$) which gave rise to losses valued at $-\Delta A_1$ in the ex-ante setting. Of course, whenever the merger decision must be taken before state realization, the possible occurence of such bad states is impounded in share price.

Turning now to a consideration of how ex-post mergers affect the individual claimant groups, assume that (1) mergers will only occur in states $s \in [s_m^\circ, s_2^\circ)$, and that (2) both projects will be exercised by the merged firm in those states. Then in the ex-post merger case, firm-1 and -2 bond values, denoted B_1° and B_2°, are

(157) $B_1^\circ = \int_{s_1^\circ}^{\bar{s}} F_1 \, q(s) \, ds$

$$(158)\quad B_2^\circ = \int_{s_m^\circ}^{\bar{s}} F_2\, q(s)\, ds$$

Bond values B_1° and B_2° correspond to rectangles adeh and acfh in Figure 6 (Chapter V). Notice that B_1° is identical to the unmerged-firm value B_1 (equation (4)). This follows from the fact that firm 1 bondholders are paid in an identical set of states by unmerged firm 1 and the ex-post merged firm. Unlike the ex-ante merger case in which firm-1 bonds decrease in value (by amount ΔB_1, equation (76)), the bonds retain their unmerged value here. Loss ΔB_1 is avoided since no merger (and subsequent default) occurs in states $s \in [s_1^\circ, s_m^\circ)$.

Firm-2 bonds, on the other hand, have the same value in the ex-post and ex-ante merger case ($B_2^\circ = B_2^m$), increasing in value relative to the unmerged value (B_2) by amount ΔB_2 (equation (77)). This increase in value is attributable to the fact that the (ex-post and ex-ante) merger brings about a coinsurance effect (which can be valued at $|\Delta e_2|$, equation (80)) as well as an improvement in project-2 investment incentives in states $s \in [s_m^\circ, s_2^\circ)$.

As shown above, the possibility of an ex-post merger increases aggregate bond value (relative to the no-merger case) by ΔB_2:

$$\begin{aligned}(B_1^\circ + B_2^\circ) - B_1 - B_2 &= (B_1^\circ - B_1) + (B_2^\circ - B_2)\\ &= \quad 0 \quad + (B_2^m - B_2)\\ &= \Delta B_2\end{aligned}$$

where $\Delta B_2 > 0$ for $s_1^\circ \neq s_2^\circ$ (Proposition V). Thus, an ex-post merger decision produces a greater bond-value appreciation than an ex-ante merger between the same set of firms. Recall that the bond value appreciation in the ex-ante case is $\Delta B = \Delta B_1 + \Delta B_2$, where $\Delta B_1 < 0$ for $s_1^\circ \neq s_2^\circ$ (Proposition V). Again, this increase in bond value (relative to the ex-ante case) is attributable to ex-post avoidance of mergers which reduce project-1 cash flows.

Even if it is rationally anticipated that ex-post mergers will only occur in states $s \in [s_m^\circ, s_2^\circ)$, market participants must formulate some expectation regarding the sharing rule to be applied to synergy gains before pricing the equity shares[49]. Assume that insiders precommit at t=0 to apportion merger gains on the basis of the Nash bargaining solution (50-50% split of ΔV°) identified in Chapter VI. Then in the ex-post case, the cum-dividend equity values (equivalently, overall levered firm values), denoted $V_1^{m\circ}$ and $V_2^{m\circ}$, are given by

$$(159)\quad V_1^{m\circ} = V_1 + G_1 = V_1 + \tfrac{1}{2}\Delta V^\circ$$

$$(160)\quad V_2^{m\circ} = V_2 + G_2 = V_2 + \tfrac{1}{2}\Delta V^\circ$$

where V_1 and V_2 are the levered firm values obtained in the absence of merger and ΔV° is the ex-post merger synergy gain specified in equation (155).

Assuming that V_1^m and V_2^m are priced (in the ex-ante case) to reflect the precommitment to the (50-50%) bargaining solution, it immediately follows that $V_1^{m\circ} > V_1^m$ and $V_2^{m\circ} > V_2^m$ by virtue of the fact that $\Delta V^\circ > \Delta V$. Specifically,

$$V_1^{m\circ} - V_1^m = (V_1 + \tfrac{1}{2}\Delta V^\circ) - (V_1 + \tfrac{1}{2}\Delta V)$$

$$= \tfrac{1}{2}(\Delta V^\circ - \Delta V)$$

$$(161)\quad V_1^{m\circ} - V_1^m = \tfrac{1}{2}\Delta A_1$$

$$V_2^{m\circ} - V_2^m = (V_2 + \tfrac{1}{2}\Delta V^\circ) - (V_2 + \tfrac{1}{2}\Delta V)$$

$$= \tfrac{1}{2}(\Delta V^\circ - \Delta V)$$

$$(162)\quad V_2^{m\circ} - V_2^m = \tfrac{1}{2}\Delta A_1$$

where $\Delta A_1 > 0$ for $s_1^\circ \neq s_2^\circ$ (Corollary 1.1).

Summarizing, in this chapter it has been shown that the ex-post merger produces greater benefits than an ex-ante merger between the same firms. Whenever an ex-post merger is possible, the utilization of the first firm's technology will be the same as that in the absence of any merger opportunity. These incentive effects for project 1 are superior to those achieved in the ex-ante case, in which project 1 is exercised in fewer states after the merger occurs. On the other hand, the incentive effects of an ex-post and ex-ante merger are identical for project 2, either of which being an improvement over the unmerged firm case. The ex-post merger thus achieves the same agency cost savings for project 2 that the ex-ante merger does, without incurring any additional loss on project 1.

CHAPTER VIII

MERGERS AND DEBT CAPACITY

In Chapter III, debt capacity was defined as the maximum amount of debt the firm can issue. In that section, debt capacity was related to the firm's technology and its all-equity value, V_o. This chapter contains an analysis of the effect of (ex-ante) mergers on debt capacity. It will be shown (Proposition 6) that mergers generally result in reduced debt capacity. In order to determine the effect of merger on debt capacity, it is useful to solve for $\overline{F}_m$, the level of promised payment which maximizes $B_m(F)$. Using the equation (20) definition of $\overline{F}$,

$$(163)\ \overline{F}_m = \tfrac{1}{2}[(b_1 + b_2)\overline{s} - (I_1 + I_2 - a_1 - a_2)]$$

Equation (20) is also useful for defining the debt-maximizing levels of promised payment of the individual firms, $\overline{F}_1$ and $\overline{F}_2$:

$$(164)\ \overline{F}_1 = \tfrac{1}{2}[b_1\overline{s} - (I_1 - a_1)]$$

$$(165)\ \overline{F}_2 = \tfrac{1}{2}[b_2\overline{s} - (I_2 - a_2)]$$

Since the equation (164) and (165) expressions for $\overline{F}_1$ and $\overline{F}_2$ sum to the equation (163) expression for $\overline{F}_m$, it is clear that

$$(166)\ \overline{F}_m = \overline{F}_1 + \overline{F}_2$$

The debt capacity of the merged firm, $B(\overline{F}_m)$, can be solved for using the equation (24) expression for $B(\overline{F})$,

$$(167)\ B(\overline{F}_m) = \frac{b_1 + b_2}{4\overline{s}}(\overline{s} - \hat{s}_m)^2$$

where $\hat{s}_m$ is defined as the state for which $V_m(\hat{s}_m) = I_1 + I_2$.

In Proposition 6 below, the merged firm debt capacity is related to the debt capacities of the individual firms, $B(\bar{F}_1)$ and $B(\bar{F}_2)$.

Proposition 6:

(a) If $\hat{s}_1 = \hat{s}_2$, then $B(\bar{F}_m) = B(\bar{F}_1) + B(\bar{F}_2)$

(b) If $\hat{s}_1 \neq \hat{s}_2$, then $B(\bar{F}_m) < B(\bar{F}_1) + B(\bar{F}_2)$

Proof:

(a) Using equation (8), $\hat{s}_m$ can be solved for as

$$\hat{s}_m = \frac{I_1 + I_2 - a_1 - a_2}{b_1 + b_2}$$

$$\hat{s}_m = \frac{I_1 - a_1}{b_1 + b_2} + \frac{I_2 - a_2}{b_1 + b_2}$$

$$(168) \quad \hat{s}_m = \frac{b_1}{b_1 + b_2}\hat{s}_1 + \frac{b_2}{b_1 + b_2}\hat{s}_2$$

From equation (168), it is clear that if $\hat{s}_1 = \hat{s}_2$, then $\hat{s}_m = \hat{s}_1 = \hat{s}_2$. The equation (167) expression for the merged firm debt capacity can then be rewritten

$$B(\bar{F}_m) = \frac{b_1}{4\bar{s}}(\bar{s} - \hat{s}_1)^2 + \frac{b_2}{4\bar{s}}(\bar{s} - \hat{s}_2)^2$$

$$(169) \quad B(\bar{F}_m) = B(\bar{F}_1) + B(\bar{F}_2)$$

(b) For $\hat{s}_1 \neq \hat{s}_2$ it will be shown that $B(\bar{F}_m) - B(\bar{F}_1) - B(\bar{F}_2) < 0$. Using the equation (167) definition of $B(\bar{F}_m)$ and equation (24) definition of $B(\bar{F})$ for $B(\bar{F}_1)$ and $B(\bar{F}_2)$,

$$B(\bar{F}_m) - B(\bar{F}_1) - B(\bar{F}_2) = \frac{1}{4\bar{s}}[(b_1+b_2)(\bar{s} - \hat{s}_m)^2 - b_1(\bar{s} - \hat{s}_1)^2 - b_2(\bar{s} - \hat{s}_2)^2]$$

$$B(\overline{F}_m)-B(\overline{F}_1)-B(\overline{F}_2) = \frac{1}{4\overline{s}}[(b_1+b_2)(-2\overline{s}\hat{s}_m + \hat{s}_m^2) - b_1(-2\overline{s}\hat{s}_1 + \hat{s}_1^2) - b_2(-2\overline{s}\hat{s}_2 + \hat{s}_2^2)]$$

Substituting the equation (168) expression for $\hat{s}_m$ into the above and simplifying yields

$$(170) \quad B(\overline{F}_m) - B(\overline{F}_1) - B(\overline{F}_2) = -\frac{b_1 b_2}{4\overline{s}\ (b_1 + b_2)} (\hat{s}_1 - \hat{s}_2)^2$$

Since b_1, b_2 and $\overline{s}$ are positive, equation (170) is negative if and only if $\hat{s}_1 \neq \hat{s}_2$. Q.E.D.

The proposition shows that the debt capacity of the merged firm is never greater than the sum of debt capacities of the unmerged firms. In the case where $\hat{s}_1 = \hat{s}_2$, the debt capacity of the merged firm is equal to the sum of debt capacities of the unmerged firms. Note that assumption (A9), which implies $\hat{s}_1 = \hat{s}_2 = 0$, is sufficient for $\hat{s}_1 = \hat{s}_2$. Whenever $\hat{s}_1 \neq \hat{s}_2$, the debt capacity of the merged firm is less than the debt capacities of the unmerged firms. This is true even though the promised payment level which maximizes merged firm debt is equal to the sum of debt-maximizing promised payment levels $(\overline{F}_1 + \overline{F}_2)$.

The proposition results are interesting as they are contrary to conclusions reached by other researchers regarding the effect of merger on debt capacity. However, as mentioned in Chapter II, other merger studies (such as Lewellan (1971), Scott (1977) and Stapleton (1982)) use definitions of debt capacity which are both different from each other and from that used here; in addition, in these studies the agency costs of debt are not considered.

<u>Corollary 6.1:</u>

A merger between two firms leveraged up to their respective debt capacities is never synergistic.

<u>Proof:</u>

Let $\overline{F}_1$ be the promised payment of firm 1 and $\overline{F}_2$ be the promised payment of firm 2. From the proof of Proposition 6 (equations (169) and (170)):

$$(171)\quad B(\overline{F}_m) - B(\overline{F}_1) - B(\overline{F}_2) \leq 0$$

From equation (66), let ΔB denote the merger-induced increase in aggregate bond value. Thus, equation (171) becomes $\Delta B \leq 0$. From Corollary 3.1, $\Delta B = \Delta V + |\Delta E|$. Therefore, $\Delta B \leq 0$ implies $\Delta V \leq 0$. Q.E.D.

The corollary shows that there is no conflict between the Corollary 3.1 result that synergistic mergers imply bond value increases and the Proposition 6 result that debt capacity is never increased by merger.

The Proposition 6 result that the maximum amount of debt which can be raised by a firm goes down or remains the same after a merger should not be interpreted as being inconsistent with the empirical regularity that the post-merger level of debt is often higher than the pre-merger level of debt (Kim and McConnell (1977)). Implications about the pre-merger and post-merger optimal levels of debt should only be derived from a model where the level of debt is endogenously determined.

CHAPTER IX

EMPIRICAL IMPLICATIONS

The major theme of the analysis is that when investment is a discretionary choice made by insiders of firms with risky debt outstanding, a merger could improve the investment incentives, leading to an increase in value. An immediate empirical question which arises is whether there is a larger incidence of mergers by firms in industries characterized by technologies which require large discretionary investments. For example, technologies requiring small initial outlays relative to heavy future discretionary investments (such as research, exploration, development or maintenance expenditures) might fall into this category.

Assuming that firms have access to zero net present value projects in the capital markets, Corollary 2.1 indicates that mergers between firms with different debt ratios (debt capacity utilization ratios) would be synergistic. Since under this corollary, most firm combinations appear to be beneficial, the frequency of mergers in the economy should be consistently high. However, even though a merger would result in a net reduction in the agency costs of underinvestment, the merger may not be implemented for various reasons. First, merger gains may be small relative to the transaction costs which must be incurred in order to complete the merger (e.g., costs associated with proxy contests or tender offers.) Second, if the outstanding debt has not been priced in full anticipation of the merger and its positive effects, a synergistic merger would result in an upward revaluation in aggregate bond value, thereby reducing the synergy gains which accrue to equity holders. Moreover, even in the cases where equity holders capture some (or all) of the merger synergy gains, the insiders of the individual firms may not be able to agree on an exchange ratio which both sides consider fair. On the other hand, as discussed at the end of Chapter VI, the synergy gain allocations documented in the empirical studies are not far from those identified using game-theoretic solution concepts.

Corollaries 1.3 and 1.5 indicate that acquiring firms should be characterized by higher debt ratios (debt capacity utilization ratios) than the firms they acquire. An indirect implication of these corollaries is that the lower the debt ratio (debt capacity utilization ratio) of the firm, the more likely it is to be an acquired firm. Therefore, in any random sample of firms, the average debt ratio (debt capacity utilization ratio) of the acquired firms should be lower than the average debt ratio (debt capacity utilization ratio) of the other firms in the sample.

Several empirical studies indicate that (1) acquired firms are less highly levered than non-acquired firms, and (2) acquired firms are less highly levered than acquiring firms. In univariate tests involving two leverage measures, Melicher and Rush (1974) find that for both conglomerate and nonconglomerate mergers, acquiring firms are significantly more levered than their merger partners. In Carleton, Guilkey, Harris, Stewart (1983), univariate and logit analysis results indicate that acquired firms use less debt than either nonacquired or acquiring firms. Stevens (1973) finds that leverage[50] is the most significant indicator of whether a firm is acquired or nonacquired in both univariate tests and a multiple discriminant analysis model. Similarly, Wansley, Roenfeldt and Cooley's (1983) use of multiple discriminant analysis to develop a financial profile of the acquired firm indicates that acquired firms generally have less debt than nonacquired firms.

Since a firm's debt capacity is not readily observable, direct empirical measurement of a firm's debt capacity utilization ratio would be difficult. However, in the context of the model, both the debt ratio and the debt capacity utilization ratio are surrogates for the s° value. As a matter of fact, Corollaries 1.2 and 1.4 imply that $D_1 < D_2$ if and only if $U_1 < U_2$ (if and only if $s_1^\circ < s_2^\circ$). Therefore, the cited studies can also be interpreted as being consistent with Corollary 1.5 implications where the studies' leverage ratios are proxying for debt capacity utilization ratio.

For a given firm, the higher the technology parameter b, the higher the cash flow and net cash flow in each state of the world. The technology parameter b can thus be related to probitability or growth measures for the firm. In a comparison of two firms which are technologically identical in all respects except for parameter b, the firm with the higher b will have higher cash flow and net cash flow in each state of the world; alternatively, the firm with the higher b would be expected to have higher profitability or growth measures than the other firm. Holding other parameters constant, b is inversely related to s° ($\partial s^\circ/\partial b < 0$); therefore, firms with low s° (acquired firms) should be characterized with high b (profitability, growth) while firms with high s° (acquiring firms) should be characterized with low b (profitability, growth).

Several empirical studies provide evidence consistent with the above predictions. For a sample of conglomerate mergers, Melicher and Rush (1974) show that acquired firms are significantly more profitable than acquiring firms along four profitability measures. Carleton, et. al. (1983) demonstrate that acquired firms are more profitable than nonacquired firms. The discriminant analysis results in Wansley, Roenfeldt and Cooley (1983) indicate that acquired firms grow more rapidly than nonacquired firms (here, growth is defined based on the growth in sales in the 3 years preceding merger).

If the debt is priced ignoring the possibility of merger, there are two effects on bond values: (1) in synergistic mergers, the combined price of bonds increases relative to the sum of their pre-merger values (Corollary 3.1), and (2) in all mergers which affect investment incentives, the value of bidding firm's bonds will increase while the value of the target firm's bonds will decrease (Proposition 5). However, since equity holders lose under the bond mispricing scenario (Proposition 3), insiders will avoid all mergers unless strategies can be devised and implemented which eliminate wealth transfers to bondholders. Empirical studies by Kim and McConnell (1977) and Asquith and Kim (1982) indicate that neither acquiring nor target firms bondholders experience abnormal gains (or losses) around the time of merger. Eger (1983), on the other hand, finds that bidder bondholders experience abnormal gains and bidding equity holders experience essentially zero abnormal returns around the merger announcement date. With the exception of Eger's bondholder results, these findings could be interpreted as being consistent with either the first scenario (in which bonds are properly priced) or with the second scenario, where countermeasures allow equity holders to recapture synergy gains and wealth transfers that would otherwise accrue to bondholders. One such measure would be the issuance of additional debt following the merger; such post-merger increases in leverage have been documented by Kim and McConnell (1977).

APPENDIX A

EXTENSION OF MODEL TO FIRMS WITH EXISTING ASSETS

The model of the firm delineated in Chapter III can be modified to include "existing" or "seizable" assets (i.e., assets whose value does not depend on future discretionary investments) without affecting the basic model result that firms with risky debt choose suboptimal investment policies. As shown below, equity holders become more eager to invest when a refusal to do so results in forfeiture of existing assets. However, investment incentives are still distorted in states which yield positive net cash flows to the firm, but negative cash flows to the equity holders.

Retain assumptions (A1) to (A8), except for a relaxation of the (A4) stipulation that technology V(s) is the only asset the firm possesses. The firm will now be assumed to be in possession of an additional asset which requires no future investment: namely, cash in the amount $K/(1 + r_f)$ at $t = 0$, which is invested in the riskless asset to yield K at $t = 1$. (Recall from assumption (A7) that the riskless rate is assumed to be zero, so that $K/(1 + r_f) = K \int_0^{\bar{s}} q(s)\, ds = K$.) In the case of the levered firm, K may have been raised from debt proceeds at $t = 0$; otherwise, K will be assumed to have been contributed by equity holders at $t = 0$.

The optimal investment rule of the all-equity firm with the cash asset is the same as that for the firm without the cash asset: invest in all positive net-present-value projects (equivalently, invest if $s > \hat{s}$). Paralleling the equation (1) all-equity valuation, the new all-equity value, denoted V_o^+, is

$$(\text{A.1})\ V_0^+ = \int_0^{\bar{s}} K\ q(s)\ ds + \int_{\hat{s}}^{\bar{s}} [V(s) - I]\ q(s)\ ds$$

The all-equity value V_0^+ is proportional to the sum of areas of triangle prx and rectangle abcd in Figure 13.

With or without the addition of assets K to the firm, it is easy to show that the underinvestment problem is nonexistent for firms with riskless debt. A sufficient condition for the debt to be riskless is that existing assets K exceed the promised payment F. Therefore, we will assume in the analysis below that $F > K$ in order to preserve the riskiness of debt.

Consistent with the limited-liability analysis set forth in Chapter III, debt holders and equity holders divide $t = 1$ cash flows according to the following sharing rules. Debt holders receive $\text{Min}\ \{F, C(s) + K\}$ and equity holders receive $C(s) + K - \text{Min}\ \{F, C(s) + K\}$, where $C(s)$ is the cash flow resulting from the investment decision. (That is, $C(s) = V(s)$ if investment I is made; otherwise, $C(s) = 0$ for all $s \in [0, \bar{s}]$.)

If the investment is passed over, the return to equity holders is thus

$$(\text{A.2})\ K - \text{Min}\ \{F, K\} = K - K = 0$$

since, as argued above, risky debt is characterized by $K < F$. If the investment is undertaken, the <u>net</u> return to equity holders is

$$(\text{A.3})\ -I + V(s) + K - \text{Min}\ \{F, V(s) + K\}$$

Since zero can always be obtained by refusing to invest, insiders will only invest if the corresponding investment return (equation (A.3)) yields more than zero:

$$-I + V(s) + K - \text{Min}\ \{F, V(s) + K\} \geq 0$$

The Investment Decision of the All-Equity Firm

with Existing Asset K

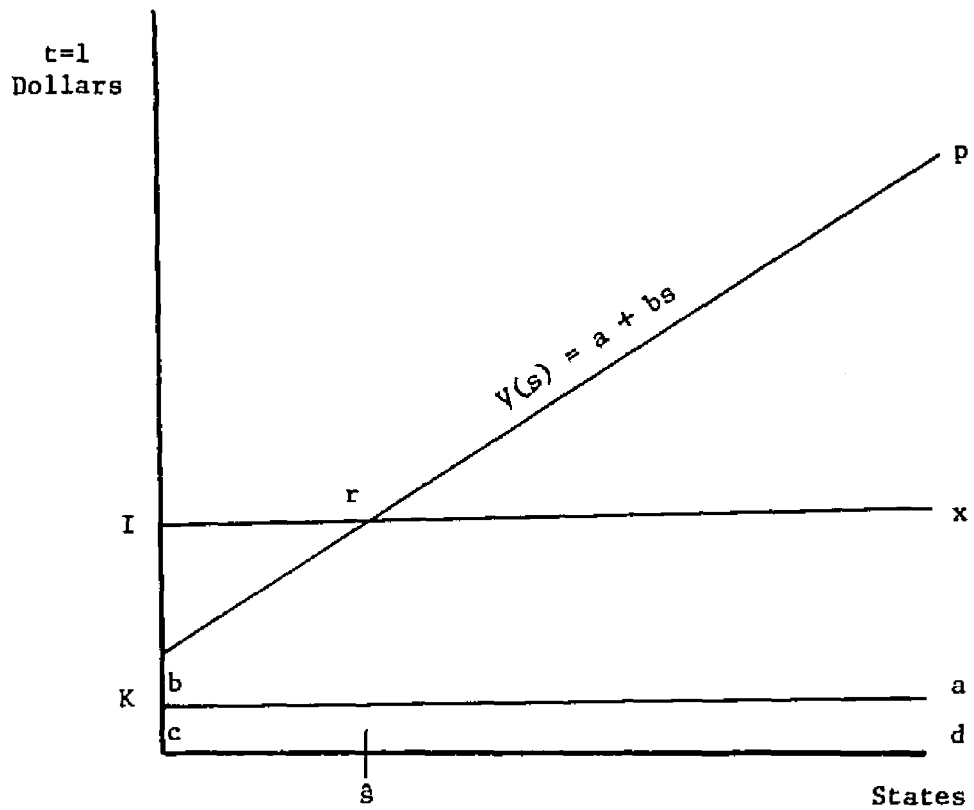

FIGURE 13

$$V(s) + K - I + \text{Max}\ \{-F, -V(s) - K\} \geq 0$$

$$\text{(A.4)} \quad \text{Max}\ \{V(s) - I - F + K, -I\} \geq 0$$

As argued earlier in Chapter III, for $I > 0$, "invest and default" always pays less than noninvestment. In other words, Max $\{V(s) - I - F + K, -I\} \geq 0$ if and only if $V(s) - I - F + K \geq 0$. Then the investment decision rule specified in equation (A.4) can be equivalently stated as invest only if

$$\text{(A.5)} \quad V(s) - I - F + K \geq 0$$

Define state s^+ such that $V(s^+) = I + F - K$. Then the investment is now undertaken in the levered firm with existing assets K for $s \geq s^+$.

Since $V(s)$ is monotonically increasing in s and $I + F > I + F - K > I$ (for I, F, and K greater than zero and $F > K$), it must be $s^\circ > s^+ > \hat{s}$, as depicted in Figure 14. That is, the investment incentives are better in the levered firm with seizable assets than in the levered firm without such assets, and better still in the all-equity firm. Specifically, levered firm investment incentives are improved over the interval of states $s \in [s^+, s^\circ)$ by the addition of cash flow K to the firm's asset profile. This follows from the fact that K accrues to equity holders if they invest (and repay F), increasing an (otherwise) negative return of $V(s) - I - F < 0$ to a positive return of $V(s) - I - F + K > 0$ for $s \in (s^+, s^\circ)$.

Having established that the investment will now be undertaken for $s \geq s^+$, it is easy to specify debt and equity claim values, denoted B^+ and E^+, respectively, as follows:

$$B^+ = \int_0^{\bar{s}} [\min\ \{F, C(s) + K\}\ q(s)]\ ds$$

The Investment Decision of the Levered Firm

with Existing Asset K

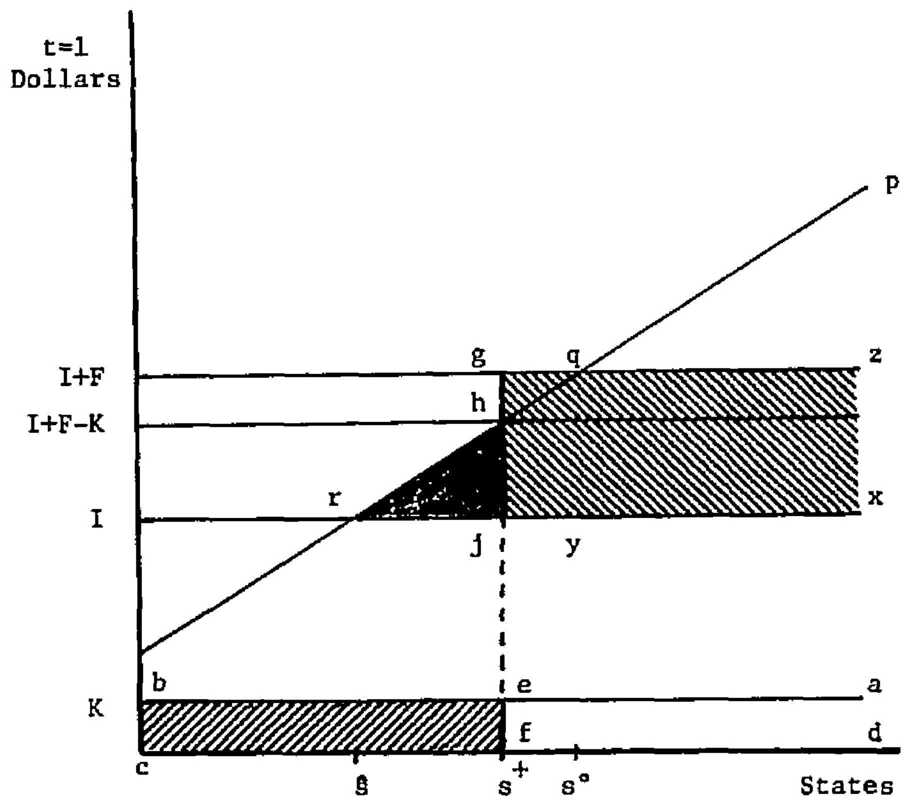

FIGURE 14

$$B^{+} = \int_{0}^{s^{+}} [\min\{F, K\}]q(s)\,ds + \int_{s^{+}}^{\bar{s}} [\min\{F, V(s) + K\}]q(s)\,ds$$

$$\text{(A.6)}\quad B^{+} = \int_{0}^{s^{+}} K\,q(s)\,ds + \int_{s^{+}}^{\bar{s}} F\,q(s)\,ds$$

$$E^{+} = \int_{0}^{\bar{s}} [\max\{C(s) + K - F, 0\}]q(s)\,ds$$

$$E^{+} = \int_{0}^{s^{+}} [\max\{K - F, 0\}\,q(s)]ds + \int_{s^{+}}^{\bar{s}}[\max\{V(s) + K - F, 0\} - I]q(s)ds$$

$$E^{+} = \int_{0}^{s^{+}} 0\,q(s)\,ds + \int_{s^{+}}^{\bar{s}} [V(s) - I - F + K]\,q(s)\,ds$$

$$\text{(A.7)}\quad E^{+} = \int_{s^{+}}^{\bar{s}} [V(s) - I - F + K]\,q(s)\,ds$$

Bond value B^{+} is proportional to the sum of the areas of cross-hatched rectangles ebcf and zgjx in Figure 14. Equity value E^{+} is proportional to the sum of areas of triangle pqz and rectangle aefd, minus the area of (cross-hatched) triangle qgh (where, but for the addition of cash flow K, equity holders receive negative cash flow $V(s) - I - F < 0$).

The levered firm value, denoted V^{+}, is equal to

$$V^{+} = E^{+} + B^{+} = \int_{0}^{s^{+}} K\,q(s)\,ds + \int_{s^{+}}^{\bar{s}} [V(s) - I + K]\,q(s)\,ds$$

$$\text{(A.8)}\quad V^{+} = \int_{0}^{\bar{s}} K\,q(s)\,ds + \int_{s^{+}}^{\bar{s}} [V(s) - I]\,q(s)\,ds$$

Levered firm value is thus proportional to the sum of areas of trapezoid phjx and rectangle abcd in Figure 14. By comparing levered and unlevered firm values represented in Figures 13 and 14, it is apparent that there is a residual loss in firm value corresponding to shaded triangle hrj in Figure 14.

By subtracting levered firm value V^{+} (equation (A.8)) from unlevered firm value V_o^+ (equation (A.1)), the residual loss in value, denoted A^+, can be computed for the firm with assets K as

$$A^+ = V_o^+ - V^+ = \int_0^{\bar{s}} K\, q(s)ds + \int_{\hat{s}}^{\bar{s}}[V(s)-I]q(s)ds - \int_0^{\bar{s}} K\, q(s)ds - \int_{s^+}^{\bar{s}}[V(s)-I]q(s)ds$$

$$\text{(A.9)}\quad A^+ = \int_{\hat{s}}^{s^+} [V(s) - I]\, q(s)\, ds$$

where A^+ is proportional to shaded triangle hrj in Figure 14. The residual loss is the agency cost of underinvestment, which exists for any levered firm with risky debt in its capital structure and some investment-contingent assets in its asset structure.

Comparing the agency costs A for the levered firm without assets K (equation (7)) to the agency costs A^+ for the levered firm with assets K (equation (A.9)), it is clear that agency costs have been reduced by the introduction of seizable assets. Denote this reduction in agency costs as ΔA^+, where

$$\Delta A^+ = A^+ - A = \int_{\hat{s}}^{s^+}[V(s) - I]\, q(s)\, ds - \int_{\hat{s}}^{s^\circ}[V(s) - I]\, q(s)\, ds$$

$$\text{(A.10)}\quad \Delta A^+ = \int_{s^\circ}^{s^+} [V(s) - I]\, q(s)\, ds = - \int_{s^+}^{s^\circ} [V(s) - I]\, q(s)\, ds < 0$$

The agency cost saving ΔA^+ is proportional to (cross-hatched) trapezoid qhjy in Figure 14. Holding V(s) fixed, it is clear that the agency cost savings will increase as K increases (which decreases s^+), up to the limit where agency costs are entirely eliminated ($\Delta A^+ = -A$), which is attained when the debt becomes riskless ($K = F$).

Alternatively stated, it is clear in Figure 13 that the firm is worth more when assets K are added to the (previously defined) asset profile V(s). This follows purely from the addition of assets K to the firm in the all-equity case--whereas for the levered firm, the firm is also worth more because agency costs have been reduced by ΔA^+. Also clear in Figure 14 is the fact that increased firm value translates into higher bond and equity claim values. Bond values increase from B (equation (4)) to B^+ (equation (A.6)) because (1) the debt is less risky, since in default states $s \in [0, s^+)$ asset K can be seized, and (2) bondholders rationally anticipate that insiders invest (and repay) in additional states $s \in [s^+, s°)$. Of course, bond holders are indifferent between holding claims valued at B^+ or B since, in either case, they pay a fair price (receive a zero net-present-value claim) for the debt.

Equity claim values increase from E (equation (5)) to E^+ (equation (A.7)) due to an increase of K in the investment payoff. However, equity holders "paid" a fair price for this increase, by either (1) contributing an additional $K/(1 + r_f)$ to the firm at $t = 0$, or (2) (in the case where $K/(1 + r_f)$ is raised from debt proceeds), passing over the opportunity to withdraw $K/(1 + r_f)$ from the firm (either in the form of perquisite consumption, dividend declarations or stock repurchases). Nevertheless, equity holders clearly benefit from the addition of K to the asset structure since it improves their investment behavior (increasing firm value by $-\Delta A^+$). Similar to Jensen and Meckling's (1976) argument re: motivation for partial sale of the firm, it must be assumed that an alternative (personal) use of $K/(1 + r_f)$ more than compensates equity holders for the related increase in agency costs.

It has been shown that adding realism through an allowance for existing firm assets is unnecessary insofar as it only complicates the claim valuation without changing the basic result that levered firms underinvest. Therefore, the simplifying assumption (that V(s) is the firm's sole asset) is retained for clarity in the merger analysis in the body of the paper. Nevertheless, it can be shown that the merger analysis in Chapters IV, V and VI can be easily extended to mergers between firms with existing assets; this involves substituting new investment threshold states s_1^+, s_2^+ and s_m^+ (where s_m^+ is defined such that $V_m(s_m^+) = \frac{I_1 + I_2 + F_1 + F_2 - K_1 - K_2 - a_1 - a_2}{b_1 + b_2}$) for s_1°, s_2° and s_m° in all merger computations. The central result that mergers reduce agency costs of underinvestment (Corollary 2.1) is thus relevant for mergers between firms with existing assets.

APPENDIX B

NECESSARY AND SUFFICIENT CONDITIONS FOR A "BOTH OR NEITHER" INVESTMENT POLICY

In general, the merged firm may exercise one of its investment opportunities while allowing the other to lapse. From the shareholders' standpoint, it will be optimal to do so if and only if

(B.1) $V_1(s) - I_1 \geq F_1 + F_2$

(B.2) $V_2(s) - I_2 < 0$

where, without loss of generality, the (more profitable[51]) firm-1 option is exercised and the firm-2 option is allowed to lapse. Condition (B.1) states that the exercised investment must have a net return which at least covers the combined promised payment, $F_1 + F_2$. Condition (B.2) states that, at the same time, the project being allowed to lapse is a negative net present value project.

Let s^* denote the threshold state in which the cash flows from the firm-1 investment just cover its investment outlay I_1 and the combined debt obligation $F_1 + F_2$. That is,

(B.3) $V_1(s^*) - I_1 = F_1 + F_2$

In addition, let $V_2(s^*) - I_2 < 0$ for purposes of the present discussion so that it would not be profitable to simultaneously exercise the second project if state s^* is realized at $t = 1$. (This assumption will later be dropped when we specify $s^* > s_m^\circ$.)

The investment choice being faced by insiders of the merged firm is depicted in Figure 15. Similar to the merged firm investment prospects depicted in Figure 3, firm 1 (with promised payment F_1)

The Merged Firm's Single-Option

Investment Decision

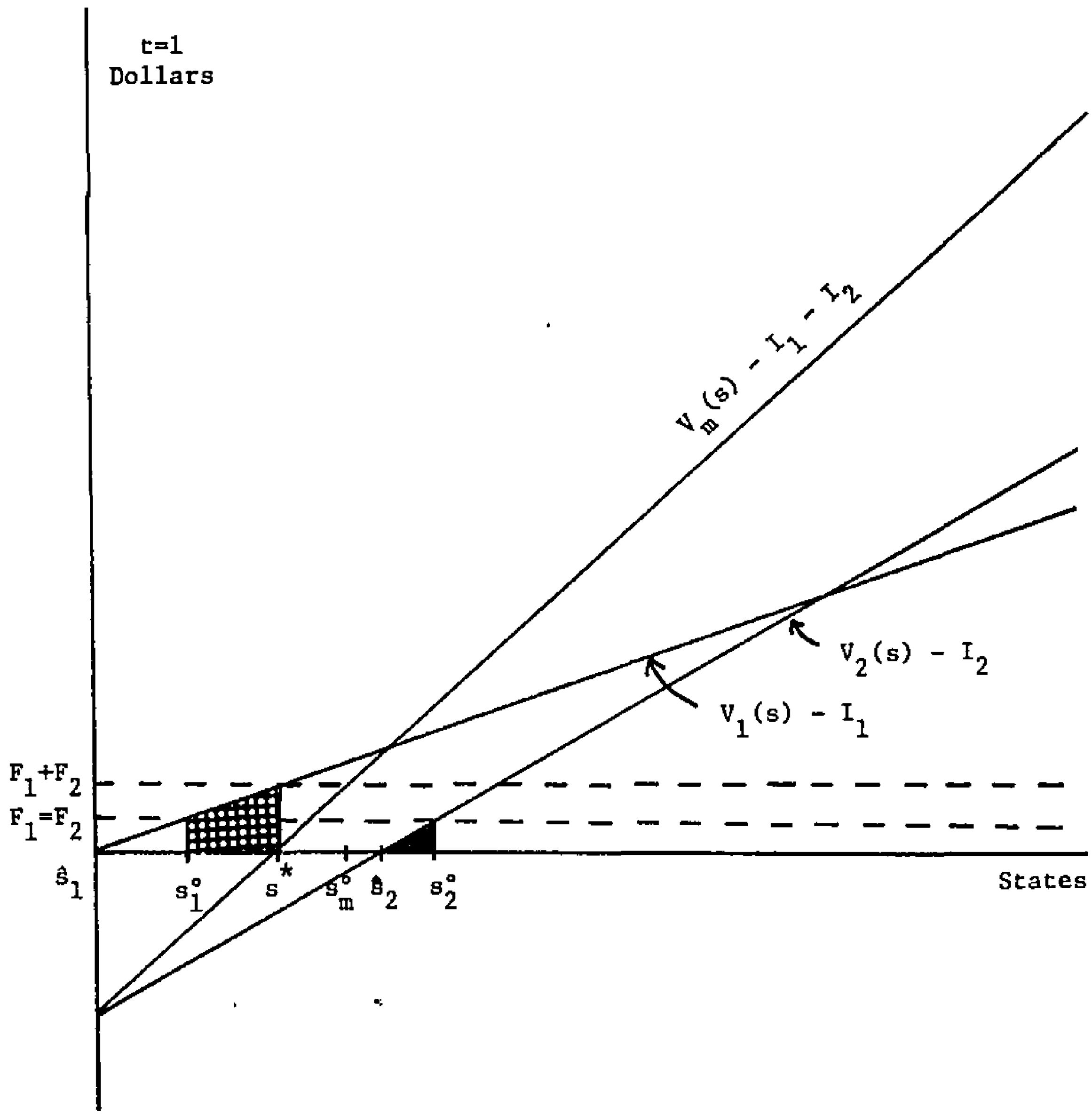

FIGURE 15

invests, standing alone, in states $s \geq s_1^\circ$; firm 2 (with promised payment F_2) invests in states $s \geq s_2^\circ$. If a merger occurs, the combined firm has access to combined technology $V_m(s) - (I_1 + I_2) = V_1(s) - I_1 + V_2(s) - I_2$. However, unlike Figure 3, the merged firm's insiders will choose to exercise only investment opportunity 1 for states $s \in [s^*, \hat{s}_2)$, which set of states satisfy equations (B.1) and (B.2). Notice that even in state s_m° (where $s_m^\circ < \hat{s}_2$), the second project will not be exercised since it yields negative net cash flows. That is, while $V_m(s_m^\circ) = I_1 + I_2 + F_1 + F_2$ (as always), $s_m^\circ < \hat{s}_2$ such that $V_2(s_m^\circ) - I_2 < 0$; the exercise of project 2 (in tandem with project 1) reduces the net cash flows below the amount obtainable through the (sole) exercise of project 1. In general, for the entire interval of states $s \in [s^*, \hat{s}_2)$, the net cash flow from project 1 alone can (more than) carry debt obligations, while the simultaneous exercise of the second project will only decrease overall cash flows.

For states $s \in [\hat{s}_2, \bar{s}]$, both projects will be exercised since (1) the net cash flows from the first project are able to (more than) satisfy combined debt levels (equation (B.1)), and (2) the second project contributes positive net cash flows $V_2(s) - I_2 > 0$. For states $s < s^*$, neither project will be exercised since the exercise of the first option generates negative cash flows to (merged-firm) stockholders (though not to the firm for $s \geq \hat{s}_1$), while the exercise of the second option generates negative cash flows to both the firm and stockholders.

The incentive effects of a merger of the type shown in Figure 15 become apparent through a comparison of the investment policies of merged and unmerged firms. Similar to the merger depicted in Figure 3, the investment incentives are improved for the second investment since the firm-2 investment is undertaken in additional states

$s \in [\hat{s}_2, s_2^\circ)$ by the merged firm. Related agency cost decreases correspond to the shaded triangle in Figure 15 (all agency costs are eliminated for the second project). The investment incentives are worsened for the first investment, however, since the firm-1 investment is passed over in additional states $[s_1^\circ, s^*)$ by the merged firm. Related agency cost increases correspond to the cross-hatched trapezoid in Figure 15. The profitability of such a merger would depend on the relative weight of the two opposing incentives effects. (The merger depicted in Figure 15 is not profitable since agency costs associated with the first investment increase by more than the decrease in agency costs associated with the second investment.)

The major difference between the merger depicted in Figure 3 and that depicted in Figure 15 is the degree of divergence in the profitability (i.e., state-contingent cash flows) of the unmerged firms. In Figure 3, both firms receive approximately the same level of positive net cash flow in many states so that the combined net cash flow is able to carry the combined debt "sooner" (in a poorer state) than can either project alone. In Figure 15, on the other hand, the first firm is far more profitable than the second, such that net cash flows from the firm-1 investment opportunity can satisfy the debt requirements in states $s \in [s^*, s_m^\circ)$ for which merged firm cash flows cannot. Moeover, even though the combined net cash flows can (more than) cover the combined debt for $s \geq s_m^\circ$, the second project will be passed over for all states $s < \hat{s}_2$ (including $s = s_m^\circ$) since its exercise contributes negative net cash flows in such states.

Of course, as can be seen in Figure 15, if the combined debt level is sufficiently high, we are returned to the "both or neither" choice faced by the merged firm in Figure 3. Alternatively, if the combined debt level in Figure 3 is sufficiently low, even merger between firms with similar profitability levels will yield a one-project investment policy over a small interval of poor or lower states. Therefore, the "both or neither" investment policy described for merged firms in Chapter IV is appropriate for mergers between firms whose profitability is not so widely divergent (nor debt levels so low) that the exercise of only one option would occasionally be optimal.

It can be shown[52] that equation (B.2) is satisfied if and only if $s^* < s_m^\circ$. That is,

(B.4) $s^* < s_m^\circ \rightarrow V_2(s_m^\circ) - I_2 < 0$ and $V_2(s^*) - I_2 < 0$

(B.5) $s^* > s_m^\circ \rightarrow V_2(s_m^\circ) - I_2 > 0$ and $V_2(s^*) - I_2 > 0$

where s^* is defined as the threshold state in which (more profitable) investment-1 cash flows just equal $I_1 + F_1 + F_2$ (equation (B.3)) and s_m° is defined as the state in which the combined investment cash flows just equal $I_1 + I_2 + F_1 + F_2$ (equation (28)).

If $s^* < s_m^\circ$, the merged firm is faced with the type of investment decision (depicted in Figure 15) in which it is sometimes optimal (i.e., $s \in [s^*, \hat{s}_2)$) to invest in only one of the two projects. If, on the other hand, $s^* > s_m^\circ$, the firm is faced with the "both or neither" investment decision (depicted in Figure 3) in which it is optimal to either exercise both projects (for $s \geq s_m^\circ$), or neither project (for $s < s_m^\circ$).

If $s^* > s_m^\circ$, the combined net cash flow can shoulder the debt "sooner" (i.e., in a lower state) than investment 1 can alone. Then the incentives are such that merged firm insiders find it optimal to invest in both projects for $s \geq s_m^\circ$, for the reasons specified in Chapters III and IV.

Alternatively stated, if $s^* > s_m^\circ$, conditions (B.1) and (B.2) will never be simultaneously satisfied such that it is optimal to solely exercise (more profitable) investment 1. While equation (B.1) will still be satisfied for state s^*, the second investment will also be exercised since (B.5) indicates that $s^* > s_m^\circ$ implies that the second investment is a positive net-present-value project at s^*, and even in the poorer state s_m°.

The necessary and sufficient condition for $s^* > s_m^\circ$ is shown below to be

(B.6) $s_m^\circ > \max\{\hat{s}_1, \hat{s}_2\}$

Equation (B.6) states that in state s_m°, both projects are contributing positive net cash flows to the overall investment return.

To prove that equation (B.6) is necessary for $s^* > s_m^\circ$, we will show that if equation (B.6) is not satisfied, then $s^* < s_m^\circ$, which means that for state s^*, investment 1 can cover $F_1 + F_2$ while the combined investment cannot. Suppose equation (B.6) is not satisfied. Specifically, suppose $\hat{s}_1 < s_m^\circ < \hat{s}_2$, as depicted in Figure 15. From the monotonicity of V(s) in state s, it follows that $V_2(s_m^\circ) < V_2(\hat{s}_2) = I_2$, or $V_2(s_m^\circ) - I_2 < 0$. Thus, $s_m^\circ < \hat{s}_2$ implies that the second investment is a negative net-present-value project when evaluated at $s = s_m^\circ$. State s_m° is defined such that $V_m(s_m^\circ) - I_1 - I_2 = F_1 + F_2$, which is equivalently stated $V_1(s_m^\circ) - I_1 + V_2(s_m^\circ) - I_2 = F_1 + F_2$. Since it has been shown that $V_2(s_m^\circ) - I_2 < 0$, it must be $V_1(s_m^\circ) - I_1 > F_1 + F_2$. Therefore, there is some state $s^* < s_m^\circ$ such that $V_1(s^*) - I_1 = F_1 + F_2$. We have thus shown that equation (B.6) is a necessary condition for $s^* > s_m^\circ$.

To prove that equation (B.6) is sufficient for $s^* > s_m^\circ$, we will show that if equation (B.6) is satisfied, then it must be $s^* > s_m^\circ$, since the reverse ($s^* < s_m^\circ$) involves a contradiction. If $s_m^\circ >$ max $\{\hat{s}_1, \hat{s}_2\}$, it must be $s_m^\circ > \hat{s}_2$. From the monotonicity of V(s) in state s, it follows that $V_2(s_m^\circ) > V_2(\hat{s}_2) = I_2$, or $V_2(s_m^\circ) > I_2$. Now suppose $s^* < s_m^\circ$. Again, from the monotonicity of V(s) in state s, it follows that $V_1(s_m^\circ) > V_1(s^*) = I_1 + F_1 + F_2$, or $V_1(s_m^\circ) > I_1 + F_1 + F_2$. Adding $V_2(s_m^\circ) > I_2$ and $V_1(s_m^\circ) > I_1 + F_1 + F_2$ yields $V_1(s_m^\circ) + V_2(s_m^\circ) > I_1 + I_2 + F_1 + F_2$, or $V_m(s_m^\circ) > I_1 + I_2 + F_1 + F_2$. This is a contradiction since, by definition $V_m(s_m^\circ) = I_1 + I_2 + F_1 + F_2$.

It has thus been shown that equation (B.6) is necessary and sufficient for $s^* > s_m^\circ$, or the "both or neither" investment policy adopted by all merged firms in the body of the paper. As discussed above, the imposition of a "both or neither" investment policy is not particularly restrictive since it will naturally follow from mergers between firms which (1) are not widely divergent in profitability prospects, and/or (2) have somewhat heavy overall debt levels.

APPENDIX C

COMPUTATION OF NASH'S BARGAINING SOLUTION

The bargaining solution to the merger game presented in Chapter VI involves finding the outcome $(\overline{V}_1^m, \overline{V}_2^m)$ which maximizes the objective function $(V_1^m - V_1)(V_2^m - V_2)$, subject to the feasibility constraints

(C.1) $V_1^m \geq V_1$

(C.2) $V_2^m \geq V_2$

(C.3) $V_1^m + V_2^m \leq V_1 + V_2 + \Delta V$

These constraints are used to form the feasible set S depicted in Figure 11. The merger bargaining solution is thus a constrained maximization with Lagragian equation

(C.4) $$L = (V_1^m - V_1)(V_2^m - V_2) + \lambda(\Delta V + V_1 + V_2 - V_1^m - V_2^m) + \mu_1(V_1^m - V_1) + \mu_2(V_2^m - V_2)$$

The bargaining solution can be found by satisfying the following Kuhn-Tucker optimality conditions:

(C.5) $\frac{\partial L}{\partial V_1^m} = (V_2^m - V_2) - \lambda + \mu_1 = 0$

(C.6) $\frac{\partial L}{\partial V_2^m} = (V_1^m - V_1) - \lambda + \mu_2 = 0$

(C.7) $\Delta V + V_1 + V_2 - V_1^m - V_2^m \geq 0$

(C.8) $V_1^m - V_1 \geq 0$

(C.9) $V_2^m - V_2 \geq 0$

(C.10) $\lambda \geq 0$

(C.11) $\mu_1 \geq 0$

(C.12) $\mu_2 \geq 0$

(C.13) $\lambda(\Delta V + V_1 + V_2 - V_1^m - V_2^m) = 0$

(C.14) $\mu_1(V_1^m - V_1) = 0$

(C.15) $\mu_2(V_2^m - V_2) = 0$

The following characterizations of the solution vector $(\overline{V}_1^m, \overline{V}_2^m)$ are collectively exhaustive and mutually exclusive:

(C.16) $V_1^m > V_1$, $V_2^m > V_2$

(C.17) $V_1^m = V_1$, $V_2^m > V_2$

(C.18) $V_1^m > V_1$, $V_2^m = V_2$

(C.19) $V_1^m = V_1$, $V_2^m = V_2$

It will be shown below that the characterization contained in equation (C.16) implies a unique solution $(\overline{V}_1^m = V_1 + \frac{1}{2}\Delta V,\ \overline{V}_2^m = V_2 + \frac{1}{2}\Delta V)$. Further, by a process of elimination, it will be shown that the bargaining solution must be drawn from equation (C.16). Outcomes described in equations (C.17) and (C.18) involve an inconsistency in the Kuhn-Tucker conditions, while the solution described in equation (C.19) does not maximize the objective function $(V_1^m - V_1)(V_2^m - V_2)$.

Consider the set of outcomes described in equation (C.16): $V_1^m > V_1$ and $V_2^m > V_2$ such that the merger benefits both bidding and target firm shareholders. If $V_1^m > V_1$, $V_2^m > V_2$, then the complementary slackness conditions in equations (C.14) and (C.15) imply

(C.20) $\mu_1 = 0 \qquad \mu_2 = 0$

Substituting the above results into equations (C.5) and (C.6) yields

(C.21) $\lambda = V_2^m - V_2 = V_1^m - V_1$

Since (by assumption) $V_1^m - V_1 > 0$ and $V_2^m - V_2 > 0$, it is clear from equation (C.21) that

(C.22) $\lambda > 0$

Thus, using the complementary slackness condition in equation (C.13),

(C.23) $\Delta V + V_1 + V_2 = V_1^m + V_2^m$

which means that the bargaining solution lies along the pareto optimal frontier, as expected.

Recall from Chapter VI that $V_i^m = V_i + G_i$ $(i = 1, 2)$ so that equations (C.21) and (C.23) can be further simplified to

(C.24) $\lambda = G_1 = G_2$

(C.25) $\Delta V = G_1 + G_2$

Equations (C.24) and (C.25) uniquely solves for G_1 and G_2 as

(C.26) $G_1 = G_2 = \frac{1}{2}\Delta V$

Substitute the above result into $V_i^m = V_i + G_i$ $(i = 1, 2)$ to determine the bargaining solution:

(C.27) $\overline{V}_1^m = V_1 + \frac{1}{2}\Delta V$

(C.28) $\overline{V}_2^m = V_2 + \frac{1}{2}\Delta V$

It has thus been shown that among all (V_1^m, V_2^m) values which leave both merger parties better off, only those described in equations (C.27) and (C.28) satisfy the Kuhn-Tucker optimality conditions specified for the Nash bargaining solution.

It will now be argued that the optimum solution cannot be drawn from the set of (V_1^m, V_2^m) points satisfying either equation (C.17) or (C.18). Any such point will trigger a contradiction in the Kuhn-Tucker optimality conditions. Suppose a solution is to be drawn from the set of points described by equation (C.17). That is, only firm-2 shareholders benefit from the merger, while firm-1 shareholders are left unaffected by merger. Since $V_1^m = V_1$, equation (C.6) can be rewritten

$$(C.29)\quad \lambda = \mu_2$$

Given the assumption that $V_2^m > V_2$, the complementary slackness condition of equation (C.15) specifies

$$(C.30)\quad \mu_2 = 0$$

Substituting $\mu_2 = 0$ into equation (C.29) yields

$$(C.31)\quad \lambda = 0$$

If $\lambda = 0$, equation (C.5) can be written

$$(C.32)\quad V_2^m - V_2 = -\mu_1$$

Since (by assumption) $V_2^m - V_2 > 0$, equation (C.32) can be rewritten

$$-\mu_1 > 0$$

$$(C.33)\quad \mu_1 < 0$$

Notice that the finding $\mu_1 < 0$ contradicts the equation (C.11) requirement that $\mu_1 \geq 0$.

Using arguments parallel to those made in equation (C.29) to (C.33), it is easy to show that the equation (C.18) specification $V_1^m > V_1$, $V_2^m = V_2$ also leads to a contradiction in the Kuhn-Tucker conditions. Thus, an optimal solution is not contained in either the set of points described by equation (C.17) or (C.18).

Finally, Nash's bargaining solution cannot be equal to the disagreement outcome described in equation (C.19). The objective function equals zero when evaluated at $V_1^m = V_1$, $V_2^m = V_2$, which is lower than the value attainable by choosing any point in the set

$V_1^m > V_1$, $V_2^m > V_2$. Moreover, it has been demonstrated that the solution described in equations (C.27) and (C.28) uniquely satisfy the Kuhn-Tucker optimality conditions in that set. It has thus been shown that the Nash bargaining solution to the merger bargaining game described in Chapter VI and depicted in Figure 11 is $\overline{V}_1^m = V_1 + \frac{1}{2}\Delta V$ and $\overline{V}_2^m = V_2 + \frac{1}{2}\Delta V$.

FOOTNOTES

[1]See, for example, Halpern (1973), Mandelker (1974), Ellert (1976), Dodd and Ruback (1977), Langetieg (1978), Kummer and Hoffmeister (1978), Bradley (1980), Dodd (1980), Jarrell and Bradley (1980), Asquith (1983), Asquith, Bruner and Mullins (1983), Bradley, Desai and Kim (1983), Eckbo (1983), Malatesta (1983), and Ruback (1983). Many of these studies address issues other than the abnormal returns obtainable by merger participants.

[2]Notable exceptions are found in Dodd (1980) and Malatesta (1983), who document negative abnormal returns to successful bidding firms.

[3]As Lewellan (1971) shows, the necessary and sufficient condition for reduction of default risk is not equivalent to a less-than-perfect correlation of merging firms' cash flows; he states, however, "in general, we would expect anything less than perfect inter-firm cash flow correlations to lead to a satisfaction of the requirement" (p. 537).

[4]For a formal analysis, see Higgins and Schall (1975) or Kim and McConnell (1977).

[5]These countermeasures may not be possible if the bond covenants contain "me first rules" which restrict managers from changing the capital and asset structure in a manner that improves the position of equity holders at the expense of bondholders. See Smith and Warner (1979) for more details.

[6]The difference in the statistical significance found for bondholders in the two studies could be due to sample differences and differences in methods of estimating excess returns. Asquith and Kim's sample consists of conglomerate mergers whereas Eger's sample consists of pure stock exchange mergers. The two studies make different assumptions about bondholder returns on non-trading days and use somewhat different techniques to estimate "normal" bondholder returns.

[7]In fact, Lewellan (1971) emphasizes the second effect (effect of mergers on debt capacity) to the exclusion of the first (effect of mergers on value of outstanding debt).

[8]The framework of analysis is adapted from Myers (1977).

[9]Since Arrow-Debreu securities have only non-negative terminal values, their current prices must also be non-negative.

[10]That is, given assumption (A7) of complete capital markets, it is well-known that the proper objective of insiders is to maximize firm value. For the formal arguments, see Arrow (1964) and Debreu (1959). Intuitively, insiders should maximize value accruing to equity holders since this allows them to choose any desired consumption bundle at t = 0, subject to a wealth constraint for which initial wealth has been maximized.

[11] In order for state-contingent contracts to be enforceable, (1) outsiders must be able to observe the state at $t = 1$, and (2) equity holders must place the investment amount I in escrow--otherwise, they may reneg under the protection of the limited liability sharing rule.

[12] Because of the relationship between V(F) and A(F) specified in equation (17), $V'(F) = -A'(F)$ and $V''(F) = -A''(F)$.

[13] As shown in Appendix A, the addition of cash to the firm unnecessarily complicates the model without changing basic results.

[14] Notice that assumption (A9) is sufficient for $\hat{s}_1 = \hat{s}_2$ since assumption (A9) implies $\hat{s}_1 = \hat{s}_2 = 0$.

[15] From the equation (9) expression for $s°(F)$ and the equation (20) expression for $\overline{F}$, $s°(\overline{F})$ can be written

$$s°(\overline{F}) = \frac{\frac{1}{2}[a + b\overline{s} - I] + I - a}{b}$$

$$s°(\overline{F}) = \frac{\frac{1}{2}[b\overline{s} + I - a]}{b}$$

$$s°(\overline{F}) = \tfrac{1}{2}[\overline{s} + \hat{s}]$$

[16] For purposes of the present discussion, assume $I_1 - a_1 = I_2 - a_2 = 0$ (under assumption (A9). An examination of the effect of firm parameters 'a' and I on ΔV then becomes uninteresting since one offsets the other, whether or not the merger occurs.

[17] Possible strategies to prevent the wealth transfer and bondholder capture of synergy gains would include the strategies mentioned in Chapter II as countermeasures to the coinsurance effect.

[18] The mild assumptions made in Appendix B are such that a "both or neither" investment policy is optimally pursued by the levered, merged firm. In the all-equity case, on the other hand, insiders maximize merged firm value by accepting all positive net-present-value projects (equivalently, invest in project i for states $s \geq \hat{s}_i$, $i = 1, 2$). It is therefore optimal to exercise only one of the two options, provided the exercised option i is a positive net-present-value project ($s \geq \hat{s}_i$), while the unexercised option j is not ($s < \hat{s}_j$).

[19] It is easy to show that the equation (69) expression for ΔB and the equation (65) expression for ΔE properly sum to ΔV (equation (50)).

[20] The coinsurance criterion thus becomes: post-merger, there is at least one state in which a deficit (negative residual cash flow) incurred on one of the investment opportunities coincides with a surplus (positive residual cash flow) in the other investment opportunity.

[21]Countermeasures can be taken under the mispricing scenario to redirect gains from bonds to equity. However, since all such defensive tactics affect the sign and magnitude of ΔE, ΔB_1, ΔB_2 in a variety of ways, their use will not be assumed here for purposes of drawing empirical implications.

[22]It will be assumed throughout the chapter that only synergistic mergers are undertaken.

[23]These assumptions are more formally defined in Axiom 1 (individual rationality) and Axiom 5 (pareto optimality), which follow.

[24]Dollar-gain merger studies, which are reviewed at the end of the chapter, typically adjust observed prices for co-movement with market prices.

[25]See Lewellan and Ferri (1984) for an exploration of the use of post-merger offer prices to infer market beliefs regarding the probability of the offer's success.

[26]To see that ΔN_2 is convex in G_1, as depicted in Figure 8, use equation (111) to compute

$$\frac{d^2(\Delta N_2)}{dG_1^2} = \frac{2N_2V_m}{(V_2 + \Delta V - G_1)^3}$$

The above is positive over the relevant ranges of values $N_2 > 0$, $V_m > 0$ and $0 \leq G_1 \leq \Delta V$.

[27]To see that X is convex in G_1, as depicted in Figure 9, use equation (115) to compute

$$\frac{d^2X}{dG_1^2} = \frac{2N_2V_m/N_1}{(V_2 + \Delta V - G_1)^3}$$

The above is positive over the relevant ranges of values $N_1 > 0$, $N_2 > 0$, $V_m > 0$ and $0 \leq G_1 \leq \Delta V$.

[28]To see that $\underline{X}$ is convex in ΔV, as depicted in Figure 10, use equation (125) to compute

$$\frac{d^2\underline{X}}{d(\Delta V)^2} = \frac{2V_1N_2}{N_1(\Delta V + V_2)^3}$$

which is positive for the relevant values of V_1, V_2, ΔV, N_1, $N_2 > 0$.

[29]Luce and Raiffa (1956), p. 121.

[30]The disagreement outcome is the expected payoff if the participants play noncooperatively (i.e., the set of maximin payoffs of the two players).

[31]Henceforth, "outcomes" and "payoffs" will be used interchangeably to refer to the vector of utilities represented by each point in S.

[32]See Roth (1979) for a more detailed treatment of the axioms and Theorem 1.

[33]I.e., the merger/no-merger decision being made by insiders of firms 1 and 2 based on the division of prospective synergy gains.

[34]Since any linear utility function is equivalent to one for which the utility of a monetary outcome \$X is equal to X, for simplicity, let $U_i(V_i^m) = V_i^m$, $i = 1, 2$.

[35]See Owen (1982) for formal proof.

[36]The normalization of (S, d) potentially involves two transformations. The first transformation obtains a disagreement outcome at the origin by setting $a_1, a_2 = 1$ and $b_1 = -d_1$, $b_2 = -d_2$ in Axiom 2. Denote the resulting game $(S°, d°)$, where $d° = (0, 0)$. The second transformation, from game $(S°, d°)$ to (S', d'), obtains $F(S', d') = (1, 1)$ without affecting the disagreement outcome $d° = d' = (0, 0)$. This can be achieved by setting $a_1 = 1/[F_1(S°, d°)]$, $a_2 = 1/[F_2(S°, d°)]$ and $b_1, b_2 = 0$ in Axiom 2.

[37]See Owen (1982) for formal proof.

[38]That is, among all equal-component vectors in A, (1, 1) is the point with the highest (equal) payoffs.

[39]See, for example, Bradley, Desai, Kim (1983).

[40]The disagreement outcome is unchanged since either firm can block a merger without impairing firm value.

[41]A more formal proof involves arguments parallel to those provided in Appendix C, after adjusting constraints to describe the new feasible set.

[42]It would be obvious that x cannot dominate y through the grand coalition.

[43]For two different axiomatic treatments of the Shapley value, see Owen (1982) and Mossin (1968).

[44]Nash (1950), pg. 158.

[45] Halpern (1973) distinguishes between firms in a merger based on the relative size of pre-merger equity values eight months prior to merger announcement. This categorization should not significantly differ from one based on the acquiring/acquired distinction since it is well-documented that acquiring firms tend to be larger than the firms they acquire.

[46] Tender offers sidetrack insiders by directly appealing to shareholders, typically offering cash-for-shares. As shown by Bradley, Desai, and Kim (1983), in a typical successful tender offer, roughly 60% of acquired-firm shares are targeted and ultimately purchased. This means the shares of a sizable minority interest may still be trading after successful execution of the offer. Several game-theoretic scenarios revolve around the relationship of this post-execution price (which doesn't exist in our 100% pure-exchange merger) to target price. See Bradley, et. al., and Grossman and Hart (1980) for details.

[47] On an abnormal <u>return</u> basis, however, the target firm typically realizes disproportionately large returns relative to the acquiring firm (see Jensen and Ruback (1983) for summary figures). This can be reconciled to gross-dollar findings with the documented regularity that acquiring firms are usually much larger than the firms they acquire.

[48] If we assume that bond proceeds are invested in the riskless asset in $t = 0$, cash available for dividends in $t = 1$ would be equal to $(B_1^\circ + B_2^\circ)(1 + r_f) = (B_1^\circ + B_2^\circ)$, where B_1° and B_2° are defined in equations (157) and (158).

[49] The same is true in the ex-ante merger case--traders form some expectation regarding the synergy gain allocation before equity claims can be priced.

[50] Both the Stevens (1973) and Wansley, Roenfeldt and Cooley (1983) studies define leverage as the book value of long-term liabilities over total assets. Similar leverage measures (involving book values rather than market values) are used in the other studies cited. Nevertheless, it is reasonable to assume that leverage measures based on book values are highly correlated with the debt ratio D, which is based on market values.

[51] Here, the "more profitable" project is used to indicate the project having net cash flows which can carry the combined debt F_1 and F_2 "sooner" (i.e., in a lower state) than the other project.

[52] If $s^* > s_m^\circ$, it must be $V_1(s^*) > V_1(s_m^\circ)$. Since $V_1(s^*) = I_1 + F_1 + F_2$, we have $I_1 + F_1 + F_2 > V_1(s_m^\circ)$. From the definition of s_m°, we have $V_1(s_m^\circ) + V_2(s_m^\circ) = I_1 + I_2 + F_1 + F_2$, or $V_1(s_m^\circ) - I_1 - F_1 - F_2 + V_2(s_m^\circ) - I_2 = 0$. Since $V_1(s_m^\circ) - I_1 - F_1 - F_2 < 0$, it must be $V_2(s_m^\circ) - I_2 > 0$. Given $s^* > s_m^\circ$, $V_2(s^*) - I_2 > 0$. Parallel arguments can be used to prove (B.4) is true.

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